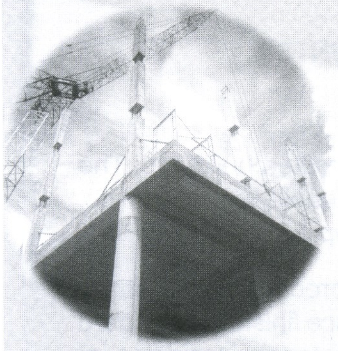


CHAPTER 2

DESIGN OF PRECAST REINFORCED CONCRETE COMPONENTS

2.1 Precast Prestressed Hollow Core Slabs

2.2 Design Of Precast Concrete Planks

2.3 Design Of Precast Reinforced Concrete Beams

2.4 Design Of Precast Concrete Columns

2.5 Design Of Precast Concrete Walls

CHAPTER 2 DESIGN OF PRECAST REINFORCED CONCRETE COMPONENTS

2.1 Precast Prestressed Hollow Core Slabs

Hollow core slabs are the most widely used precast floor component in prefabricated buildings. The success is largely due to the highly efficient automated production method, good quality surface finish, saving of concrete, wide choice of structural depths, high strength capacity and rapid assembly on site.

The hollow core slabs are manufactured using long line extrusion or slip-forming processes; the former process being the most popularly used. Cross section, concrete strength, and surface finish are standard to each system of manufacture. Other variations include increased fire resistance, provision of penetrations, opening of cores for on-site fixings, cut-outs for columns/walls, etc.

The hollow core slabs are based on 1.2 m nominal width. Most manufacturers produce the units at 1196 mm width to allow for construction tolerance. The units are available from standard depth of 150 mm to 500 mm.

The slabs are sawn after detensioning which normally takes place six to eight hours after casting and typically when the concrete strength reaches 35 N/mm^2. Non-standard units are produced by splitting the units longitudinally. Although the cut gives the rectangular plane ends as the standard, skewed or cranked ends in non-rectangular floor layout may also be specified.

Hollow core slabs are generally designed to achieve two-hour fire resistance. Fire resistance up to a maximum of four hours can be designed and produced by either raising the level of the centroid of the tendons or by increasing the concrete cover. To prevent spalling of concrete for covers exceeding 50 mm at elevated temperature, a light transverse steel mesh below the prestressing tendons is often cast at the bottom of the slabs in four-hour fire resistance units.

Holes in the floor may be created in the precast units during the manufacturing stage before the concrete has hardened. The maximum size of the opening which may be produced in the units depends on the size of the voids and the amount of reinforcement that can be removed without jeopardising the strength of the unit. Holes should preferably be located within the void size which may vary in different sections. The designer must be consulted for larger openings which involve the removal of prestressing tendons. General information on the restrictions of opening size and location is given in reference 4 and guides from the manufacturer are shown in Table 2.1 and Figure 2.1.

Depth of slab (mm)	Corner cut-out L x B (mm)	Edge cut-out L x B (mm)	End cut-out L x B (mm)	Middle cut-out L x B (mm)	Middle hole diameter ø (mm)
165	600 x 400	600 x 400	1000 x 400	1000 x 400	80
215	600 x 380	600 x 400	1000 x 380	1000 x 400	130
265	600 x 260	600 x 400	1000 x 260	1000 x 400	130
300	600 x 260	600 x 400	1000 x 260	1000 x 400	170
400	600 x 260	600 x 400	1000 x 260	1000 x 400	170

Table 2.1 Opening In Hollow Core Slabs

It is not practical to cast sockets or surface fixtures into the soffits of the precast units. These must be formed on-site. Fixing by shot fired methods is not recommended. There are limits to the maximum fixing depths at soffits in the webs of the units to prevent accidental severing of the tendons. The list of acceptable proprietary fixing anchors should be obtained from the manufacturer when planning services routing or selecting the suitable hanger system for ceilings.

In local practice, a layer of topping concrete varying from 60 to 75mm is usually included in the construction of hollow core units as structural floors. The topping thickness is generally specified at the support of the units with minimum thickness of 40mm to 55mm maintained at mid-span. The reduction of topping thickness at mid-span is due to prestressing cambers.

In wet weather, water may penetrate into the voids of hollow core slab through the open ends or surface cracks. This should be drained off before permanent floor connections are made. A simple method is to drill weep holes in the slabs at each void location, usually during the production stage.

It is common to find cracks on the surface of the precast units. These cracks may be inherent during the production stage or as a result of the handling and delivery. The types of cracks and their effects on the structural behaviour of the precast units are published by FIP and are shown in Figure 2.2. In doubtful cases, testing of the units should be carried out to verify their structural performance.

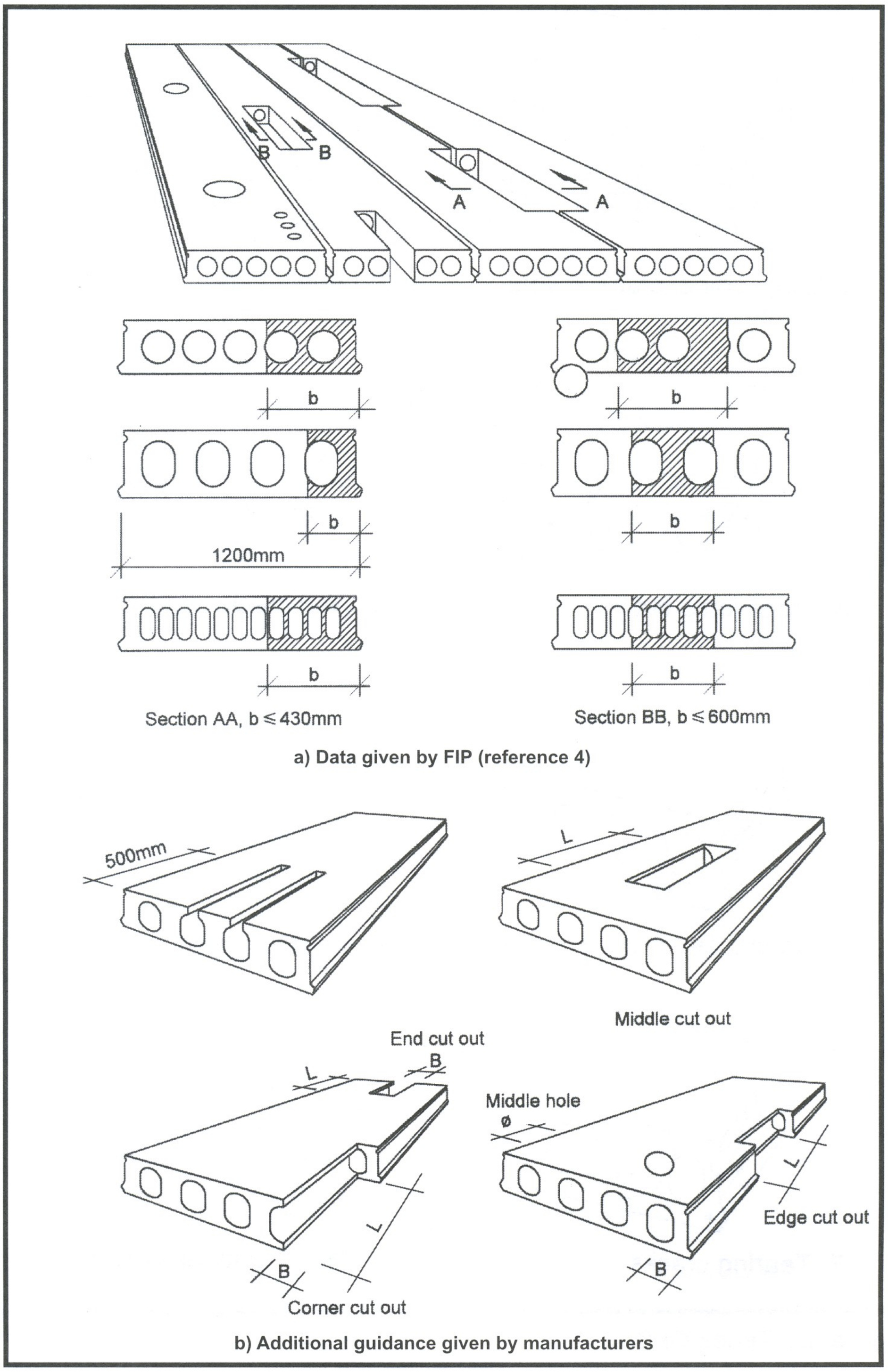


Figure 2.1 Rules For The Permitted Sizes And Locations Of Openings And Cut-Outs In Hollow Core Slabs (Also see Table 2.1)

Possible types of cracks on hollow core slabs can be categorised according to FIP report as follows:

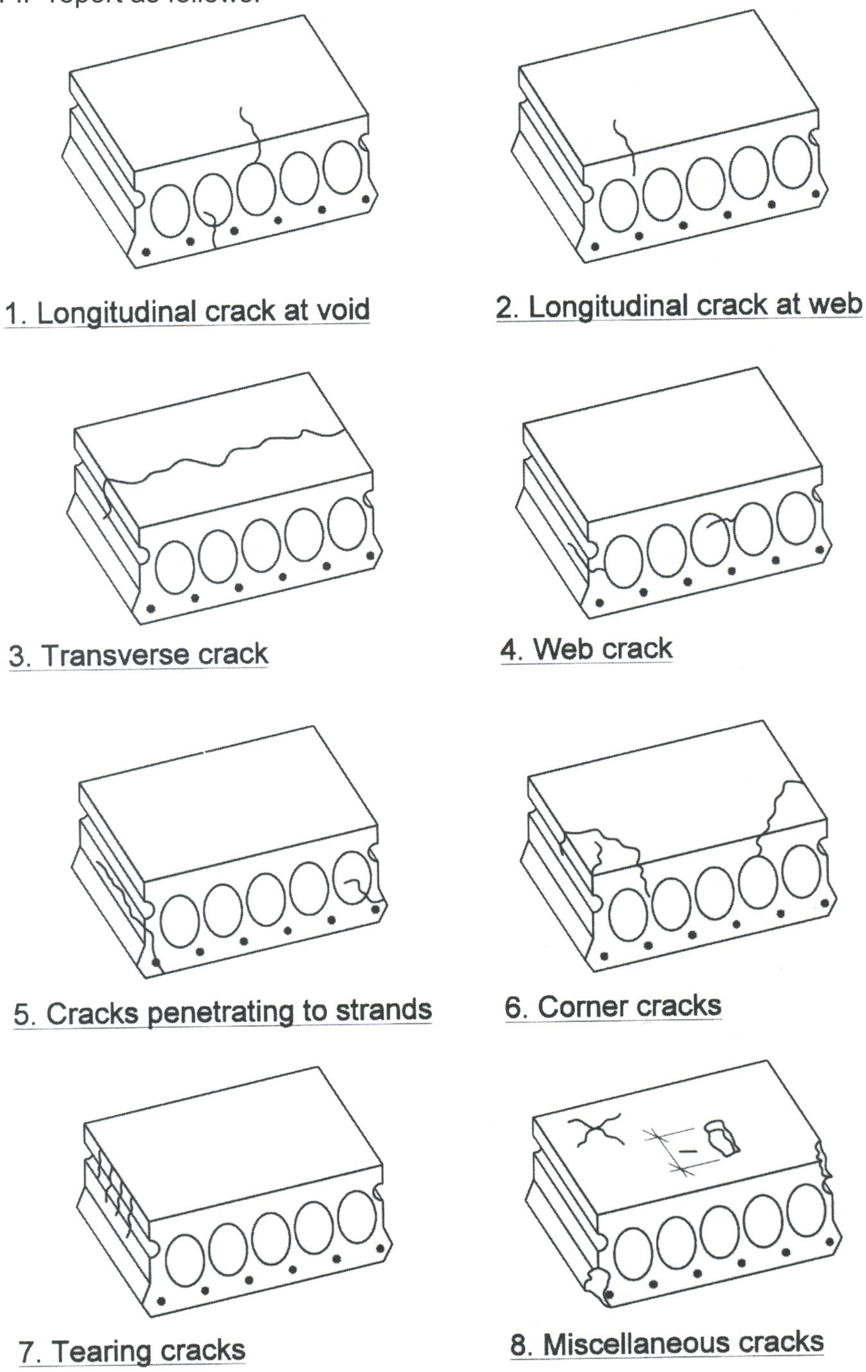


Figure 2.2 Types Of Cracks

Type	Effect on serviceability	Repair
1.	Severe or full-depth cracks in an untopped system can affect load distribution when there are concentrated loads, or transverse cantilevers. A whole-length crack at the bottom flange is dangerous at the lifting stage.	Voids can be grouted solid or the crack may be epoxied. Repair is not required if cracks are localised, not full-depth or full-length.
2.	Only little effect.	If the crack is deep, possibly penetrating up to the top strand and long, then the slabs should be used only with topping. With minor cracks, repair is not required.
3.	Potential shear capacity reduces if crack occurs at the end of the slab. It can have a significant effect on shear and moment capacities of cantilevers.	Slabs can be repaired if the crack is further away from the end than the anchorage length and is at a maximum 1/3 x depth. For minimal cracks, epoxy or solid grouted voids can be used. Minor cracks ($\leq$ 0.2 mm) in top flange at areas of positive moment or, in bottom flange at areas of negative moment, may not require any repair.
4.	It can be dangerous at the lifting stage and can reduce shear capacity.	If many of the webs are cracked, the slab should be rejected. Small single crack can be accepted.
5.	It can reduce shear capacity. Evaluation must include the effects of the associated strand slippage.	Slipped strands cause load reduction, and load bearing capacity has to be checked.
6.	Usually, a minimal effect.	Smaller cracks need not be repaired. If the corner is cracked, it should be calculated similar to opening and voids and should be grouted solid.
7.	Usually no effect.	Joint grout will automatically fill the cracks.
8.	Usually no effect.	Dropped flanges can be repaired by grouting if the damaged area is limited ($l \leq 0.5 \sim 1.0$ m)

Figure 2.2 (cont'd)

2.1.1 Design considerations for hollow core slabs

Hollow core slabs are normally produced by high strength concrete of not less than 50 N/mm^2. This is made possible by using zero slump concrete (w/c = 0.36) together with placing machineries which produce high internal pressure, shear movement compaction and high vibration energy during the extrusion process. The extrusion process makes it difficult to incorporate any other reinforcement than longitudinal prestressing tendons into the precast units. Therefore, unlike conventional reinforced or prestressed concrete structures, the strength of hollow core slabs depends on the stresses induced by prestressing force and the tension and compression capacity of the concrete.

The prestressing tendons or strands commonly used are 270k, seven-wire low relaxation (class 2) plain or indented helical strands conforming to either the BS 5896-1980 or to ASTM A416-1980 Supplement. The tendons are pretensioned to between 60 - 65% of the characteristics strength in local practice. The in-service effective prestressing force after losses from steel relaxation, creep, shrinkage and deformation is typically at 45 - 50% of the characteristics strength. The steel area is relatively low with ρ_{ps} (= A_{ps}/bd) between 0.1 to 0.25%.

The tendons are anchored by bond and are exposed at the open ends of the units. The effective pull-in of the tendons, determined from depth gauges on the centre wire of the helical strand, is typically less than 3 mm.

The design of hollow core slabs may be based either on the guidance in reference 4 or on the stipulations for general prestressed concrete design in CP65. Some important aspects specifically related to hollow core slabs design will be discussed in the following sections.

1. Design for serviceability limit state

The serviceability limit state design is based on satisfying the limits on :

- flexural tensile and compressive stresses in the concrete, and
- camber and deflection

a. Flexural tensile and compressive stresses in the concrete

The stress limitations apply to hollow core slab section at all ages and under all possible loading conditions. Most designers specify class 2 prestressed concrete structure which permits flexural tensile stresses not exceeding $0.45\sqrt{f_{cu}}$ or a maximum 3.5 N/mm^2 (C60 concrete) but without any visible crack. The flexural concrete compression is limited to $0.33f_{cu}$ at service and $0.5f_{ci}$ at prestress transfer.

Under class 2 stress limitations, the section is considered uncracked and the net cross-sectional properties are used to compute the maximum fibre stress at the top and bottom of the section.

The service moment of resistance is being the lesser of

$$M_s = (f_{bc} + 0.45\sqrt{f_{cu}}) \times Z_b$$

or

$$M_s = (f_{tc} + 0.33f_{cu}) \times Z_t$$

where $f_{bc} = P_e (1/A_c + e/Z_b)$ and
$f_{tc} = P_e (1/A_c - e/Z_t)$
f_{tc}, f_{bc} are the top and bottom fibre stresses
Z_t, Z_b are the top and bottom section moduli
e is the eccentricity of prestressing force from the geometrical neutral axis

b. **Camber and deflection**

The calculated values of camber and deflection are based on the flexural stiffness E_cI of the section, the support condition and the loading arrangement. Many variables affect the stiffness such as concrete mix especially the water/cement ratio, curing method, strength of concrete at the time of transfer and at the time of erection, relative humidity, etc. Because of these factors, the calculation of short-term and long-term camber and deflection should be treated only as estimations.

An efficient and general procedure in the calculation of camber and deflection is to determine the curvatures and then apply the curvature-area theorem. For straight tendons and simply supported members, the curvature due to prestress consists of three parts :

i. **Instantaneous curvature at transfer**

$^1/_{r_b} = P_i\, e/(E_c\, I)$ (upwards)

ii. **Due to prestress losses**

$^1/_{r_b} = \delta P\, e/(E_c\, I)$ (downwards)

iii. **Due to long-term creep effect**

$$\frac{1}{r_b}\text{(long-term)} = \phi\,\frac{[P_i + (P_i - \delta P)] \times e}{2E_cI} \quad \text{(upwards)}$$

$(1/2)[P_i + (P_i - \delta P)]$ represents the average value of prestressing force and ϕ is the creep coeffcient which can be determined from Part 2, Figure 7.1, CP65.

The total long-term curvature due to prestress is given by (i) + (ii) + (iii) as

$$\frac{1}{r_b}\text{(long-term)} = \frac{P_i\, e}{E_cI}\left[\eta + \frac{1+\eta}{2}\,\phi\right]$$

where $\eta = (P_i - \delta P)/P_i$ is the prestress loss ratio.

The curvatures due to applied load are simply :

$$\frac{1}{r_b}\text{(short-term)} = \frac{M_s}{E_cI} \quad \text{(downwards)} \qquad \text{and,}$$

$$\frac{1}{r_b}\text{(long-term)} = \phi\,\frac{M_s}{E_cI} \quad \text{(downwards)}$$

Added together, the total long-term curvatures due to applied load is

$$\frac{1}{r_b}\text{(long-term)} = \frac{1+\phi}{E_c}\,\frac{M_s}{I} = \frac{M_s}{E_{ce}I}$$

where $E_{ce} = E_c/(1 + \phi)$ is known as the effective modulus.

iv. **Total deflection**

For straight tendons in hollow core units, the total deflections can be estimated simply:

Deflection due to self weight and applied loadings:

$$\delta = \frac{5wl^4}{384E_cI} \quad \text{(downwards)}$$

Deflection due to self weight and applied loadings:

$$\delta = \frac{\eta P_i e l^2}{8E_cI} \quad \text{(upwards)}$$

An alternative approach to the estimation of long-term camber and deflection is to use the creep multipliers recommended in the PCI Design Handbook (reference 5) as shown in Table 2.2 below:

	Without composite topping	With composite topping
At Erection		
1. Deflection (downward) component – apply to the elastic deflection due to the member weight at release of prestress	**1.85**	**1.85**
2. Camber (upward) component -apply to the elastic camber due to prestress at the time of release of prestress	**1.80**	**1.80**
Final :		
3. Deflection (downward) component - apply to the elastic deflection due to the element mass weight at release of prestress	**2.70**	**2.40**
4. Camber (upward) component - apply to the elastic camber due to prestress at the time of release of prestress	**2.45**	**2.20**
5. Deflection (downward) – apply to elastic deflection due to super-imposed dead load only	**3.00**	**3.00**
6. Deflection (downward) - apply to elastic deflection caused by composite topping	**-**	**2.30**

Table 2.2 Suggested Multipliers To Be Used As A Guide In Estimating Long-term Cambers And Deflections For Typical Members

When designing camber and deflection, the following considerations need to be taken into account :

i. Aesthetic deflection limits of 1/250 is applied to units not supporting nonstructural elements which might be damaged by large deflection.

ii. When the units carry non-structural elements sensitive to large deflection, a more conservative approach is needed and guidance is given in Part 2, clause 3.2.1.2, of the Code.

iii. Transverse load distribution due to concentrated or line loads should be considered.

iv. When estimating long-term deflections, suitable levels of design load should be considered as outlined in Part 2 clause 3.3, of the Code.

2. Modes of failure and ultimate limit state design

Under increasing loads, four modes of failure of prestressed hollow core slabs may be distinguished.

a. flexural failure,
b. shear tension failure,
c. shear compression failure, and,
d. bond and anchorage failure.

a. Flexural failure

Flexural failure, shown in Figure 2.3, may occur in critical sections of maximum bending. Due to the relatively small steel area, the failure mode is characterised by flexural cracking at the slab soffits, excessive deflection and, finally, rupture in the prestressing tendons.

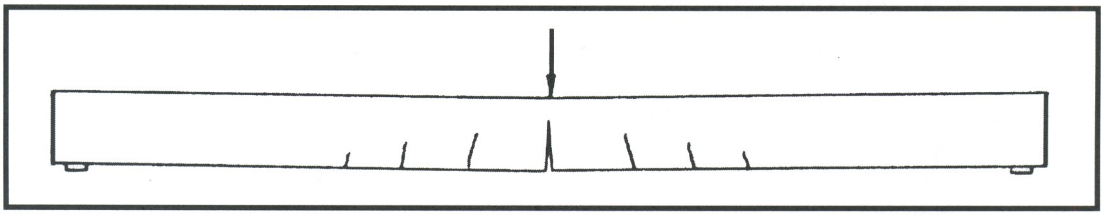


Figure 2.3 Flexural Failure

Calculations of the flexural bending capacity of a cross section can be based on the stress distribution diagram shown in Figure 2.4. In so far as χ is within the top flange thickness, the flexural capacity of the section can be calculated from Part 1, Table 4.4, BS 8110. When χ is within the void area, the value of χ can only be obtained by geometrical or arithmetic means.

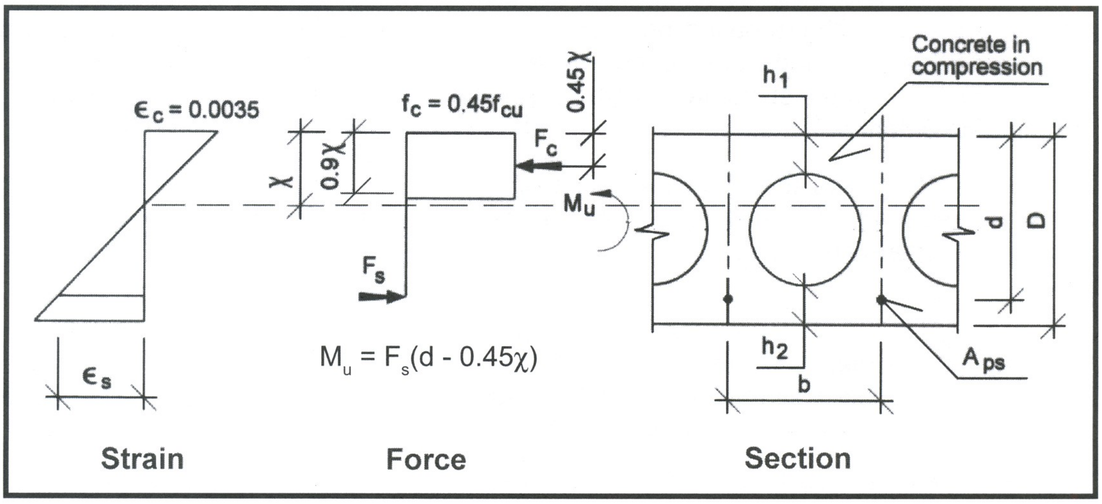


Figure 2.4 Strain And Force Distribution In Hollow Core Slab At Ultimate Limit State

b. Shear tension failure

If the principal tensile stress in the web reaches the tensile strength of concrete in an area containing no flexural cracks, an inclined crack may appear and failure may occur suddenly. The crack usually appears at the critical section where the favourable influence of support reaction is no more significant and where the prestressing force is not yet fully developed.

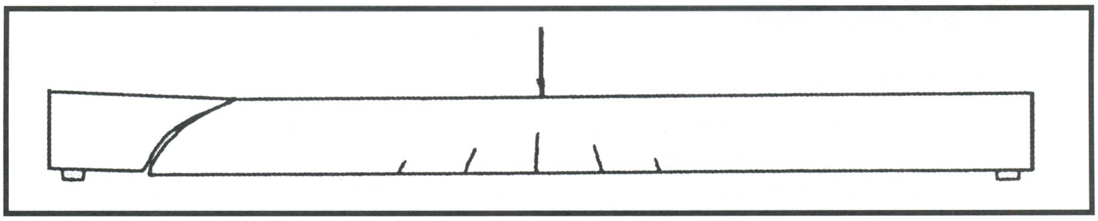


Figure 2.5 Shear Tension Failure

The existence of void in hollow core slabs complicates the theoretical calculation of stresses in the web area and it is necessary to introduce simplifications in the design method. Part 1, clause 4.3.8.4, assumes that the extremity of the support reaction spreads at an angle of 45° from the inner bearing edge. The critical section is taken as the distance from the bearing edge which is equal to the height of the centroid of the section above the soffit. The ultimate shear capacity is then calculated as:

$$V_{co} = 0.67bh\sqrt{(f_t^2 + 0.8f_{cp}f_t)}$$

where $f_t = 0.24\sqrt{f_{cu}}$

f_{cp} = concrete compressive stress at the centroidal axis due to effective prestress at the end of prestress transmission length

The expression 0.67bh is based on rectangular section and for hollow core slabs, it may be replaced Ib/S; where I and b are the respective second moment of area and breadth of the hollow core section and S the first moment of area about the centroidal axis. Ib/S usually works out to be about 0.7 to 0.8bh.

The Code recognises the fact that critical shear may occur in the prestress development length where f_{cp} is not fully developed. The prestressing force is assumed to develop parabolically according to the expression :

$$f_{cp\chi} = (\chi/l_p)(2 - \chi/l_p)\, f_{cp}$$

where χ is measured from the ends of the unit.
l_p is taken as the greater of the transmission length $K_t ø/\sqrt{f_{ci}}$ or the overall depth, D, of the member.

When the critical section is within the transmission length, the uncracked ultimate shear capacity V_{co} will need to be assessed with reduced $f_{cp\chi}$.

c. **Shear compression failure**

Shear compression failure occurs if a flexural crack develops into a shear crack which propagates through the member into the compression zone, leading to an eventual crushing of the concrete. The failure will occur most likely in the vicinity of a concentrated load near to the supports. If the load is uniformly distributed, there is a high probability that shear compression failure will not occur as the shear tension capacity near the support will normally be exhausted before shear compression becomes too excessive.

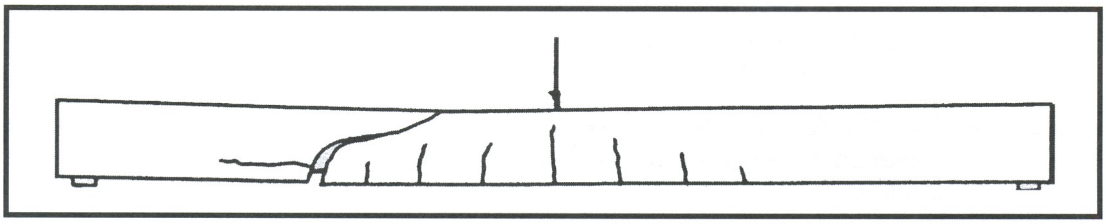


Figure 2.6 Shear Compression Failure

The design ultimate shear resistance of a flexurally cracked section is calculated using the semi-empirical equation in the CP65:

$$V_{cr} = (1 - 0.55\ f_{pe} / f_{pu})v_c\, bd + M_o V/M$$

where f_{pe} / f_{pu} is the ratio of the effective prestress after losses to the characteristics strength of the prestressing tendons.

v_c is the permissible shear stress in Part 1, Table 3.9, of the Code.

M_o is the moment necessary to produce zero stress in the concrete at the effective depth, d, level. It may be calculated as $0.8f_{pt}I/y$ where f_{pt} is the concrete compressive stress due to effective prestressing force at depth d and distance y from the centroid axis of the section which has a second moment of inertia I.

V and M are the ultimate shear force and bending moment respectively at the section considered.

In design, V_{co} and V_{cr} must always be determined and the lesser of the values governs the shear capacity of the precast unit.

d. Bond and anchorage failure

This mode of failure usually occurs when the slab is subjected to heavy concentrated loads near the support or when heavy loads are applied over a rather short span. The failure is initiated by a flexural crack resulting in a loss of bond around the prestressing tendons due to insufficient anchorage beyond the crack in the uncracked region. Figure 2.7 shows tensile stress, f_{ps}, in the prestressing tendons at point A, where the resulting flexural stress from prestressing and bending moment reaches the flexural tensile capacity of the concrete and :

$$f_{ps} = \left(\frac{1}{A_{ps}}\right)\left(\frac{M}{Z} + V\right)$$

where the additional V/A_{ps} is due to the development of direct tensile stress in the tendons resulting from shear displacement of the cracked section and Z the section modulus at the level of prestressing tendons.

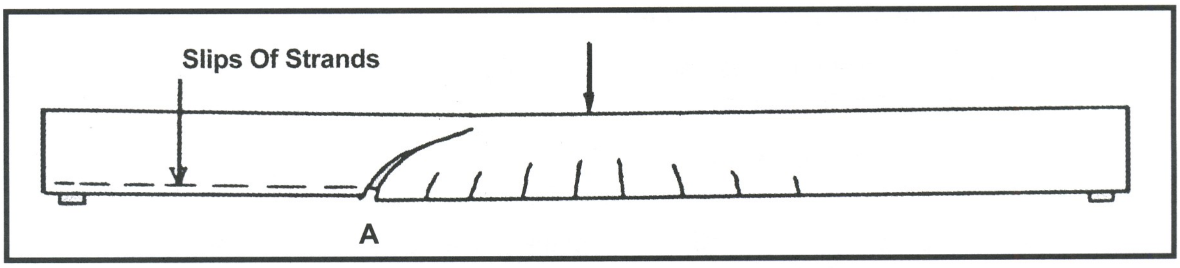


Figure 2.7 Bond And Anchorage Failure

The limiting values of f_{ps} can be determined from the anchorage failure envelope shown in Figure 2.8.

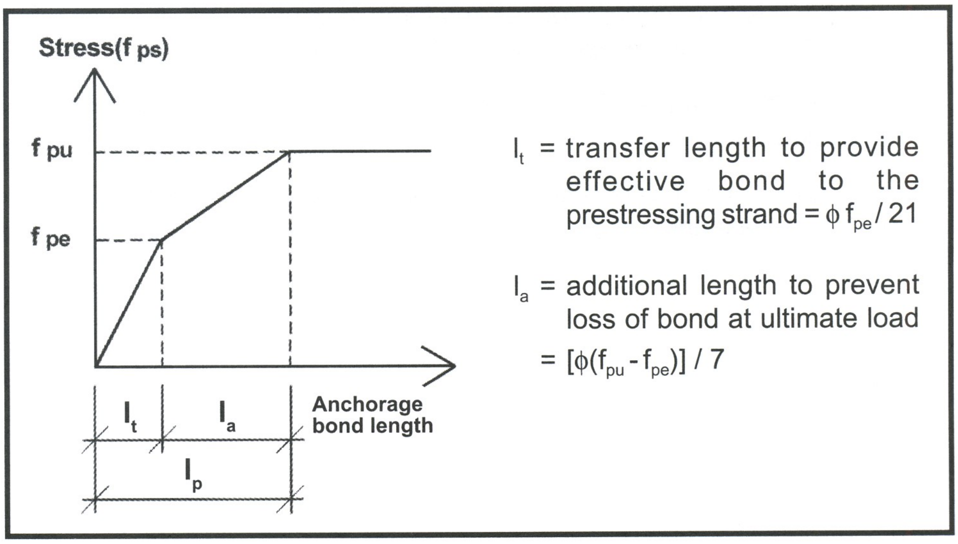


Figure 2.8 Anchorage Failure Envelope

3. Transverse load distribution and joint strength

The transverse distribution of line and point loads on precast hollow core slabs gives rise to :

a. bending moments in the transverse direction of the slab units, and
b. vertical shear forces in the longitudinal joints

Generally, hollow core slabs are manufactured without any transverse bottom reinforcement except light wiremesh for fire resistance purposes. The flexural resistance to transverse bending in the slab is hence solely dependant on the tensile strength of the concrete. From extensive field experience gathered in Europe and America, such omission of transverse reinforcement is justified in hollow core slabs.

The determination of the load distribution is generally based on tests or from theoretical analyses using the following assumptions :

a. floor units are simply supported and 1.2 m wide,
b. concentrated loading is linear and acting parallel to the span of the slab, and
c. the units are provided with transverse ties to prevent them separating from each other.

Four sets of load distribution curves are shown in the Handbook and they cover the following cases :

a. linear loading in hollow core slab floor with and without concrete topping, and
b. point loads in the span and edge of the floor without concrete topping

Alternatively, the designers may adopt the more conservative approach provided by the Code in Part 1, clause 5.2.2. It allows load distribution over an effective width which is equivalent to that of the total width of three precast units or 1/4 of the span on either side of the loaded area. For floors with reinforced concrete topping, load distribution over the total width of four precast units is permitted.

When the precast unit is designed with the entire load acting directly on it, the vertical shear in the joints need not be considered as there will be zero shear force in the transverse joint. However, when transverse load is unevely distributed or if point or line loads are present, the vertical shear capacity in the longitudinal joint will have to be assessed.

Extruded hollow core slabs have a natural random surface roughness of up to about 2 mm deep indentation. This surface is, however, classified as smooth in the Code and a design ultimate shear stress value of 0.23N/mm^2 is permitted in Part 1, clause 5.3.7a of the Code for the shear transfer at the joint. However, the value appears to be high when compared with 0.1N/mm^2 in reference 4. The designer should use the lower value of 0.1N/mm^2 when designing the average horizontal shear stress in the longitudinal joints as in reference 4 which deals with hollow core floor design specifically.

In general, the vertical and horizontal shear capacity at the transverse joints are rarely critical in design provided the joint infill is placed and well compacted. Although FIP stipulates minimum C20 concrete, local practice tends to cast the joints together with topping concrete which is normally specified with minimum C30 in design.

2.1.2 Design charts

The design charts on the selection of hollow core slabs for different loading and span are shown in Figure 2.9.

Transverse load distribution factors for line and point load on hollow core slab floor with or without topping are given in Figures 2.10 to 2.13. The actual design and disposition of prestressing tendons may be different between precast manufacturers and they should be consulted in the final design of the components.

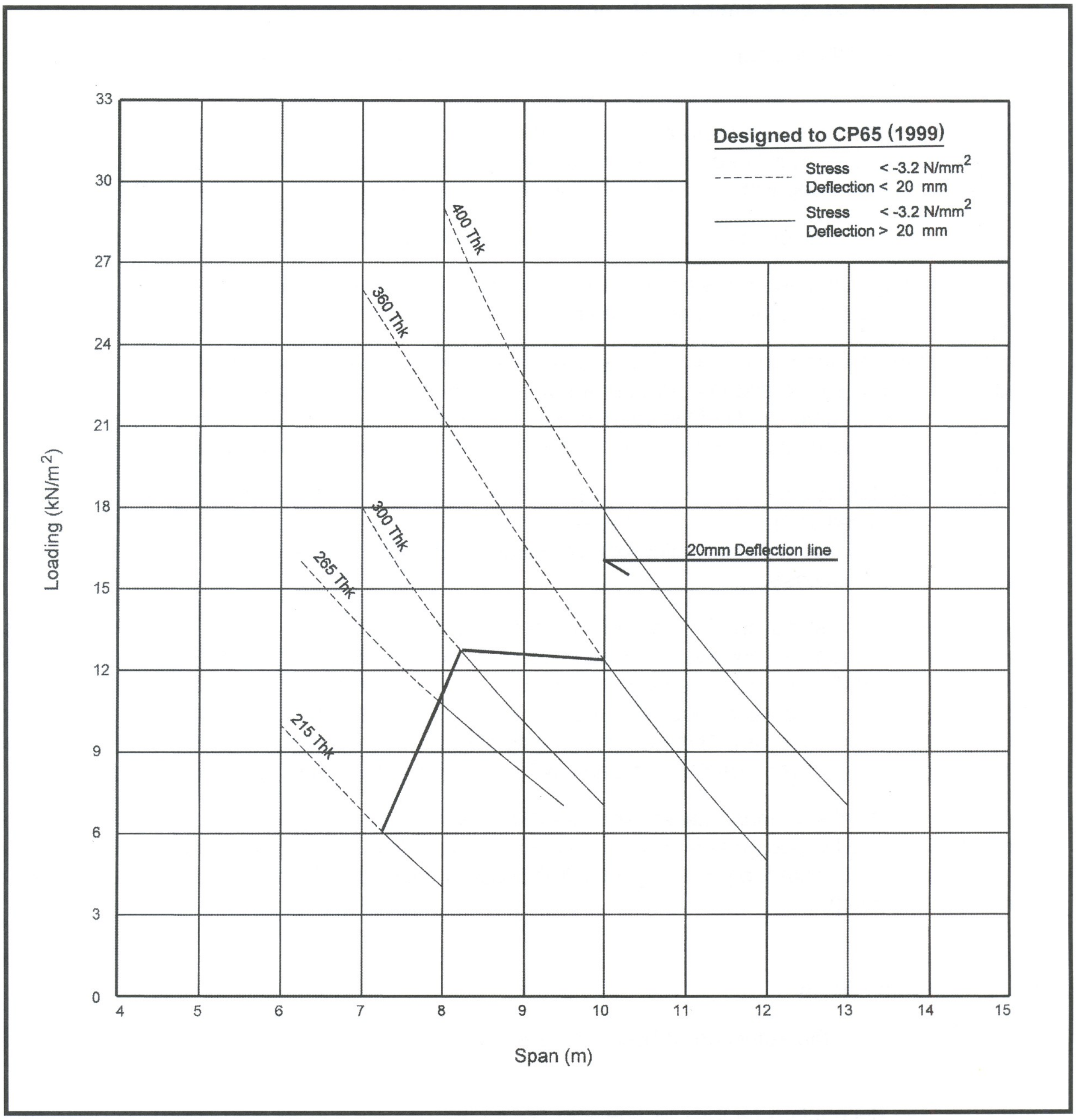


Figure 2.9 Loading Chart Based On Bending And Shear Capacity For Hollow-Core Slab

Details are intended for general information only. Precasters should be consulted for actual design of the slabs.

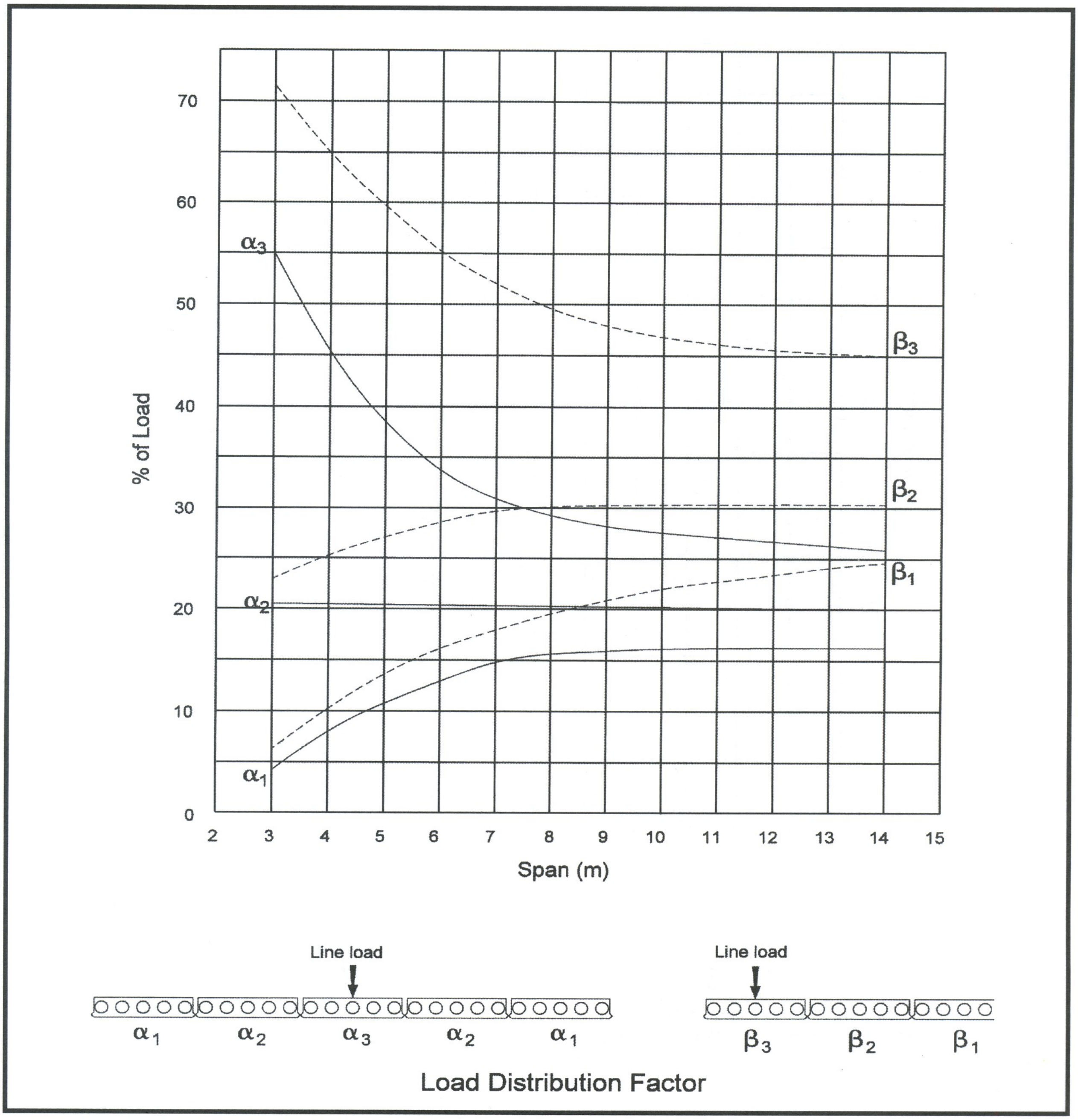


Figure 2.10 Load Distribution Factors For Linear Loadings In Hollow-Core Slab Floors Without Topping

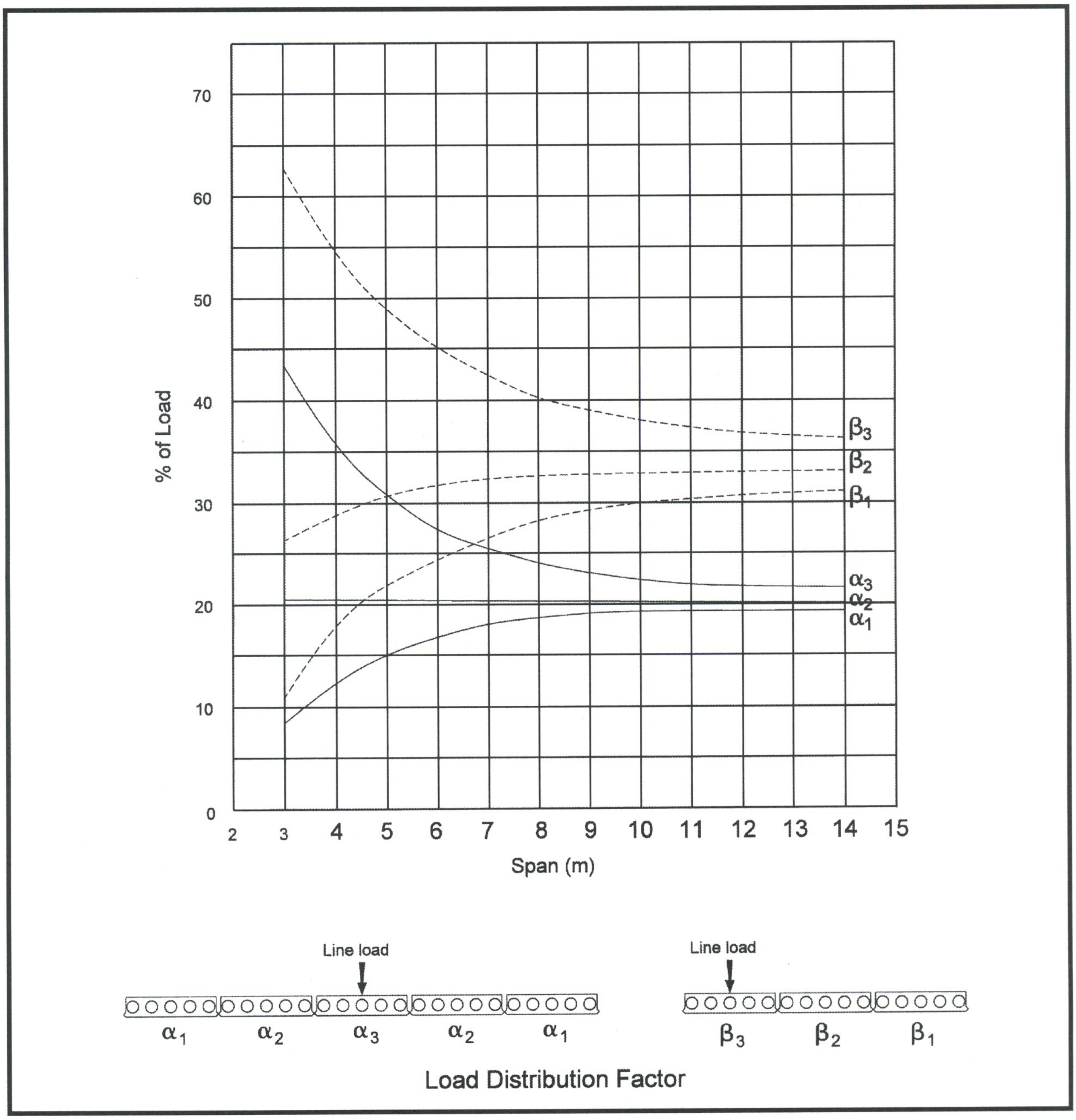


Figure 2.11 Load Distribution Factors For Linear Loadings In Hollow-Core Slab Floors With Toppings

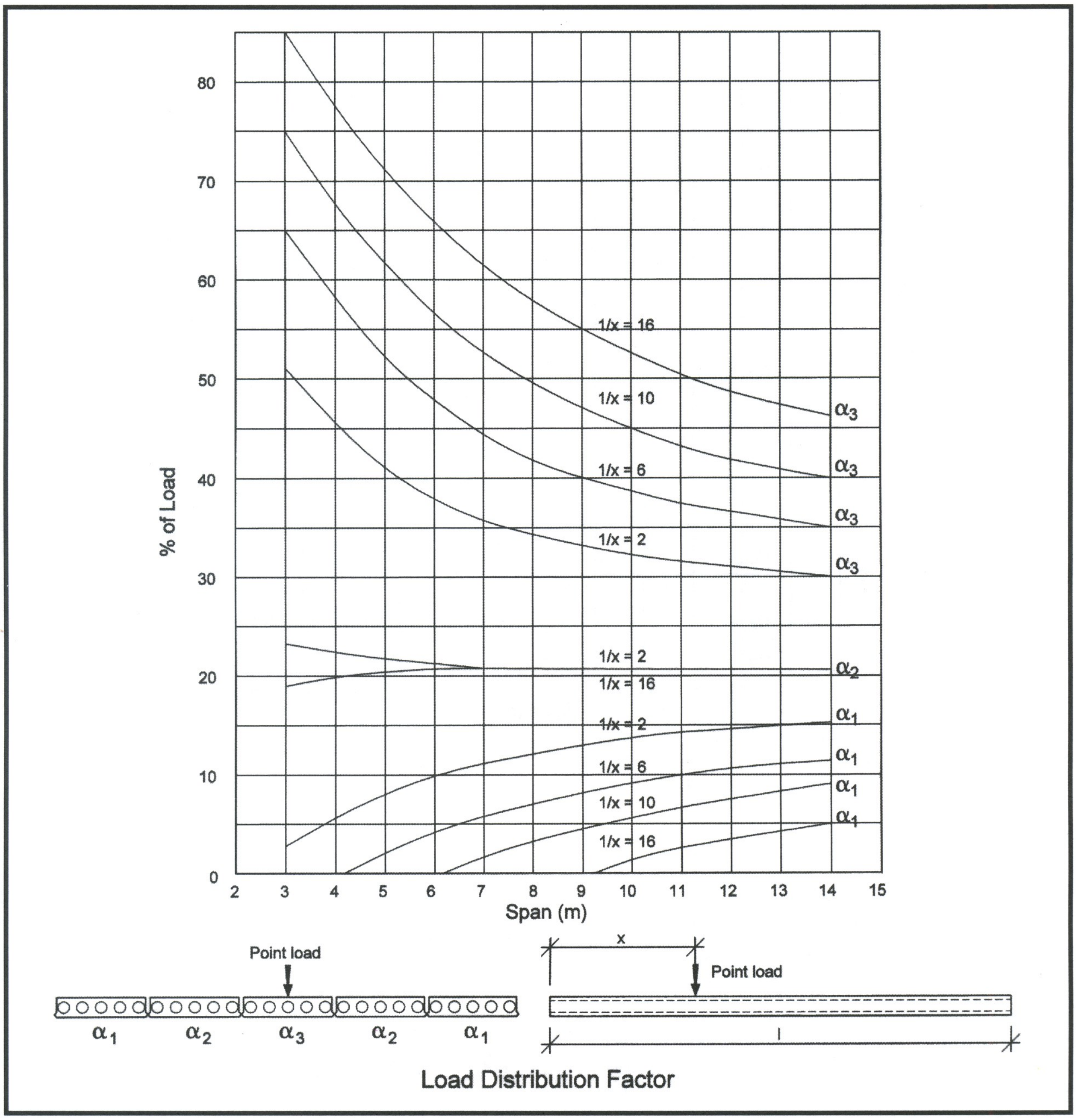


Figure 2.12 Load Distribution Factors For Point Load In The Centre Area Of Hollow-Core Slab Floors With Topping

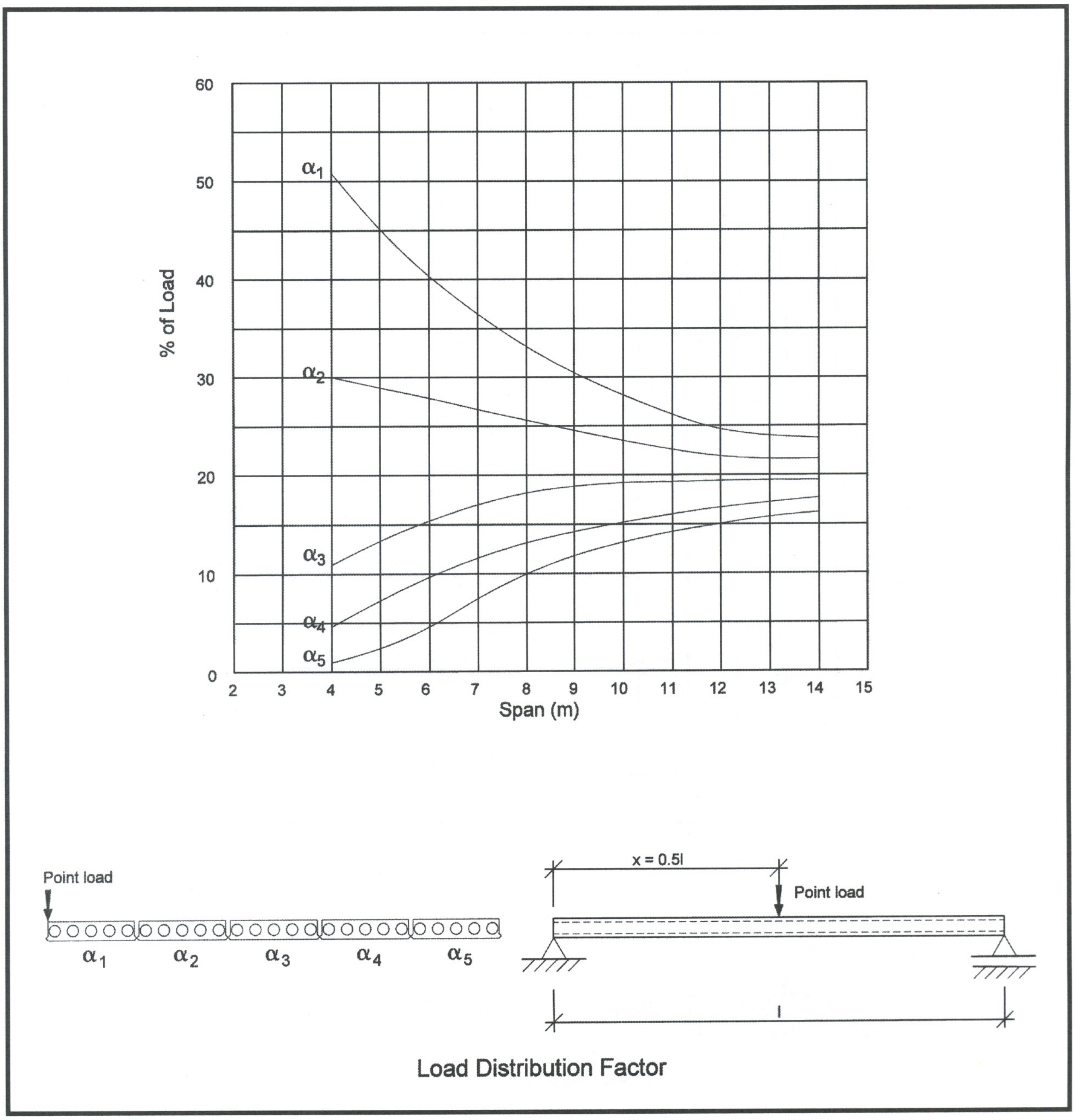


Figure 2.13 Load Distribution Factors For Mid-span Point Load At The Edge Of The Slab Field Without Topping

2.2 Design Of Precast Concrete Planks

2.2.1 General

Solid precast concrete floor planks have the advantages of smooth soffit finish, no formwork and rapid assembly at site. The planks are laid between supporting members and used as permanent formworks for in-situ topping concrete which may vary between 50 mm and 200 mm thickness. In terms of performance, the precast floors are considered equal to that of cast in-situ construction.

The precast planks are usually 75 mm or 100 mm thick although planks of 80 mm and 110 mm thick are common in the market. The choice of thickness is primarily based on :

1. construction economy, and
2. preferred method of construction

The planks can be designed either ordinarily reinforced or prestressed. Steel bars, meshes or prestressing tendons are placed in the precast units as flexural bottom reinforcement. Steel meshes are generally incorporated in the topping concrete to serve both as structural floor ties as well as negative reinforcement over the support if the slabs are designed as continuous structure. Minimum design concrete grade is generally C35 for ordinary reinforced and C40 for prestressed planks. Despite the introduction of steel meshes in the topping or in the precast units, the planks are usuallly designed as one-way spanning without utilising the two-way capabilities.

During the temporary installation stage, the planks are designed as simply supported with an unpropped span of up to a maximum of about 4 m and 6 m for 75 mm and 100 mm thick prestressed units respectively. For longer spans, intermediate props are needed. The effect of props must be taken into design considerations. The plank units are most critical when the self weight of the wet concrete topping is added to the self-weight of the plank. An allowance of 1.5 kN/m^2 of construction loads should generally be considered in checks during the temporary stage.

In the permanent condition, the hardened in-situ concrete topping provides the compression resistance and the flexural resistance in the span provided by the bottom reinforcing bars or prestressing tendons. The hogging moment resistance at the supports is provided by in-situ placed meshes or steel bars.

2.2.2 Prestressed concrete planks

Precast planks prestressed with pretensioned tendons offer the advantages of longer unpropped spans and the resistance to cracking due to handling.

The planks are usually designed to class 2 stresses. Class 3 structures are appropriate for thicker units with heavy imposed loads. The use of class 3 stresses in the plank design will facilitate detensioning of thicker units as they will be subjected to unbalanced effects due to eccentric prestressing force.

The design of prestressed planks is similar to those in the design of prestressed concrete and two design examples are presented to illustrate the various design considerations.

2.2.3 Reinforced concrete planks

The design of flexural resistance and shear follows the usual methods for reinforced concrete.

If the maximum bending moment at the temporary installation stage due to self weight and topping concrete is M_1 at a given section, the area of reinforcement is calculated as:

$$A_{s1} = \frac{M_1}{z_1 \times 0.87 f_y}$$

where z_1 is the lever arm in the precast unit.

If the ultimate moment due to imposed loads, including the effect of props, is M_2, the total area of steel at the section is given as:

$$A_s = A_{s1} + A_{s2}$$

$$= \frac{1}{0.87 f_y} \left(\frac{M_1}{z_1} + \frac{M_2}{z_2} \right)$$

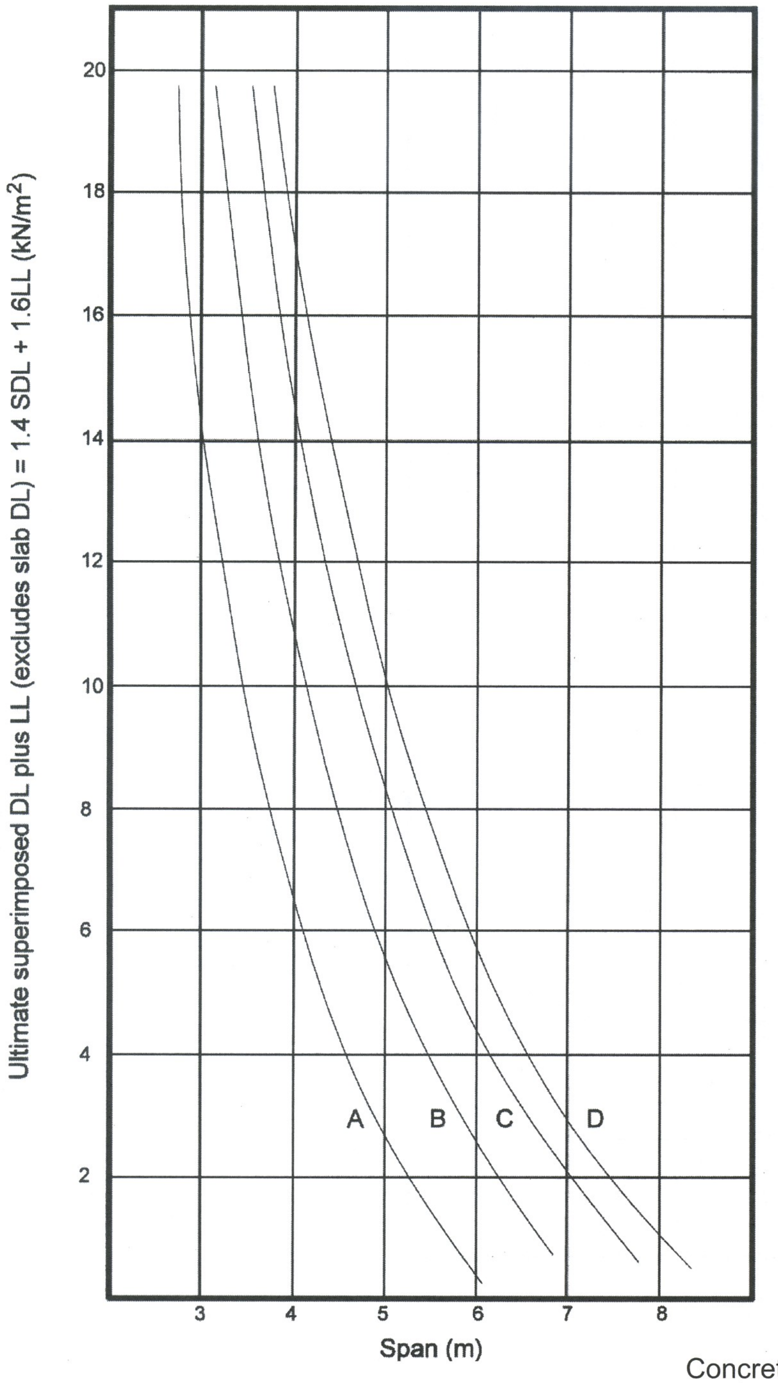
where z_2 is the lever arm in the composite section.

Vertical shear is rarely critical as there is a larger effective support width for the floor slabs. It may, however, be critical if there are openings which reduce the effective widths at the supports. Additional shear reinforcement in the form of stirrups projecting above the precast units may be provided, if necessary.

The effect of point and line loads is considered in the same manner as in cast in-situ construction. Interface shear is checked as per the code requirements in Part 1, clause 5.4.7.

2.2.4 Design charts

Two design charts, Figures 2.14 and 2.15, are presented as aids to the preliminary sizing of prestressed plank members for general design purposes.

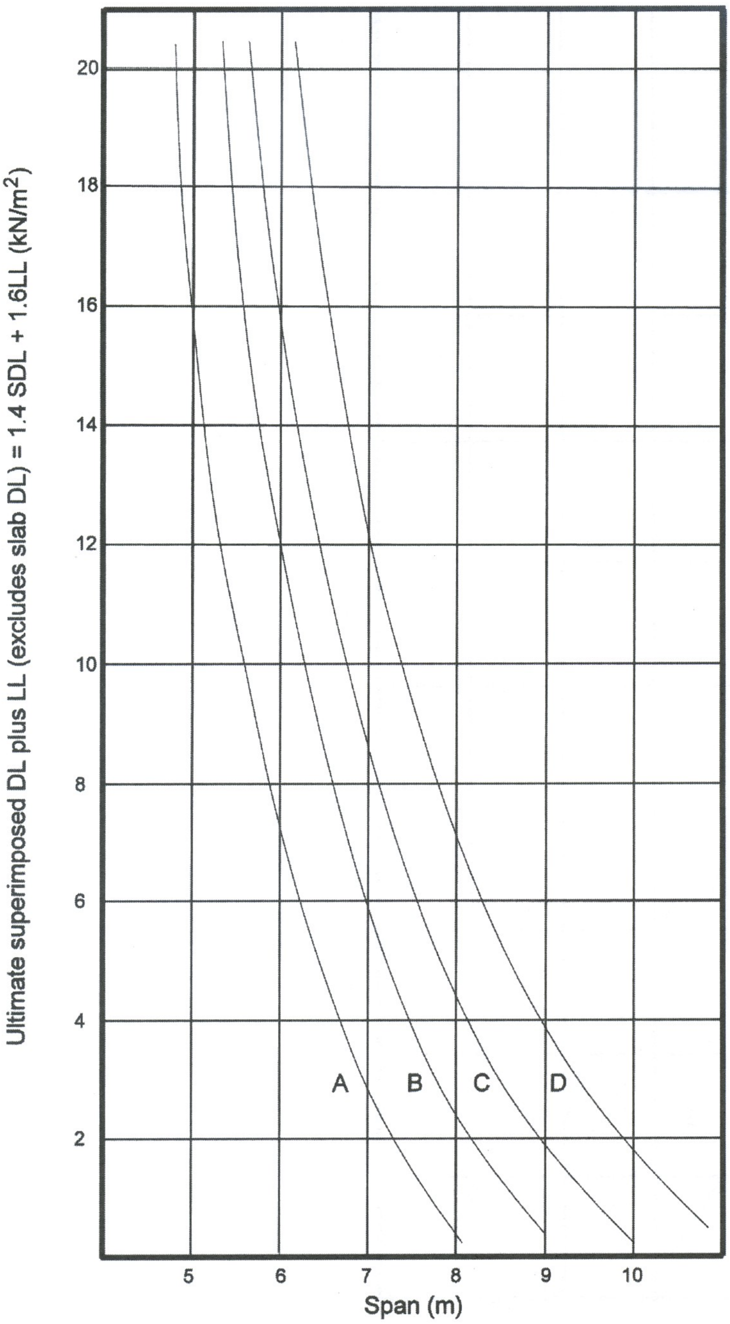


Concrete topping thickness = 55mm

Plank concrete: Grade 40
9.6mm diameter stress relieved strands conforming to BS 5896-1980, (grade 270k) at the following strand spacing:

Curve A	300mm c/c
Curve B	200mm c/c
Curve C	150mm c/c
Curve D	120mm c/c

Figure 2.14 Prestressed Solid Planks (80mm)



Concrete topping thickness = 55mm

Plank concrete: Grade 40
9.6mm diameter stress relieved strands conforming to BS 5896 - 1980, (grade 270k) at following strand spacing

Curve A	300mm c/c
Curve B	200mm c/c
Curve C	150mm c/c
Curve D	100mm c/c

Figure 2.15 Prestressed Solid Planks (110mm)

2.2.4 Design examples

Design Example 1 : Unpropped Prestressed Precast Concrete Planks

Design the prestressing reinforcement in 80 mm thick pretensioned prestressed planks with a 3.6 m simply supported span. The planks will act compositely with 65 mm thick concrete topping eventually. The design data are as follows :

1. **Design loading :**

Finishes	=	1.20 kN/m^2
Services	=	0.50 kN/m^2
Live load	=	2.00 kN/m^2

2. **Materials**

a. Prestressing tendons :
- Ultimate tensile stress f_{pu} = 1860 N/mm^2
- Initial prestress f_{pi} = 1395 N/mm^2 (75% of f_{pu})
- Prestress loss ratio (assumed) η = 0.75

b. Concrete :
- planks f_{cu} = 40 N/mm^2
- topping f_{cu} = 35 N/mm^2
- at transfer f_{ci} = 25 N/mm^2
- cover to steel c = 30 mm

3. The plank is to be designed as class 2 structure.

4. The method of construction is unpropped.

A. Service Stress Design

Step 1 : Calculate bending stresses at installation

Loading

Dead load :
- plank s/w = 0.080 x 24 = 1.92 kN/m^2
- topping = 0.065 x 24 = 1.56 kN/m^2
- Total = 3.48 kN/m^2

Live load (construction) = 1.50 kN/m^2

Moment at mid-span

a. Due to self weight, M_1 = 3.48 x 3.6^2/8
= 5.64 kNm/m

Top and soffit concrete stress in plank f_c = $\pm$ 6M_1 / bh^2
= $\pm$ (6 x 5.64 x 10^6) / (1000 x 80^2)
= $\pm$ 5.28 N/mm^2

b. Due to s/w and construction live load M_1 = (3.48 + 1.50) x 3.6^2/8
= 8.07 kNm/m

Top and soffit concrete stress in plank f_c = $\pm$(8.07 x 10^6 x 6) / (1000 x 80^2)
= $\pm$ 7.56 N/mm^2

Step 2 : **Calculate bending stresses at service**

Loading

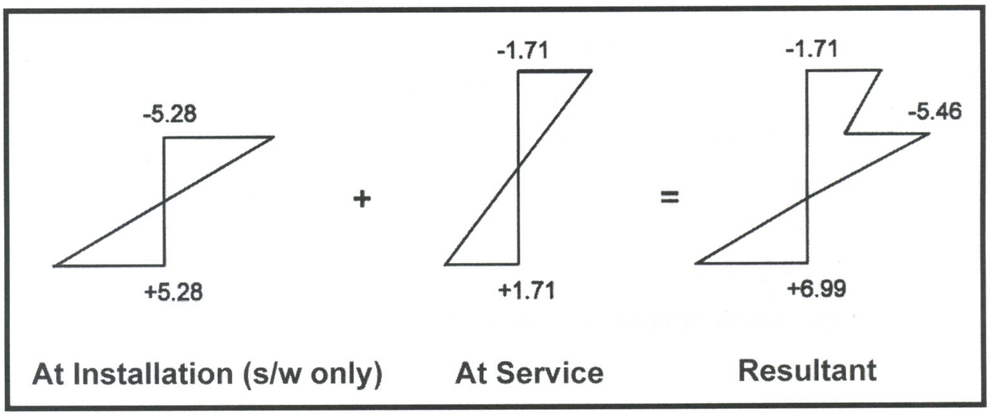
Dead load :	finishes	=	1.20 kN/m^2
	services	=	<u>0.50 kN/m^2</u>
			1.70 kN/m^2
Live load		=	<u>2.00 kN/m^2</u>
Total		=	3.70 kN/m^2

At mid-span, moment M_2 = $3.70 \times 3.6^2/8$
= 6.00 kNm/m

Top and soffit concrete stress in plank f_c = $\pm (6.00 \times 10^6 \times 6) / (1000 \times 145^2)$
= $\pm$ 1.71 N/mm^2

Step 3 : **Resultant bending stress**

At mid span :



Note : +ve denotes concrete tensile stress
-ve denotes concrete compressive stress

Step 4 : Calculate effective prestressing force and reinforcement

Permissible tensile stress (class 2) at plank soffit (Part 1, clause 4.3.4.3)

$f_t = 0.45\sqrt{f_{cu}}$
$= 2.8$ N/mm^2

Ignoring eccentricity effect of prestressing ($e \approx 5$ mm), the required effective prestressing force in the plank is

$P_e = (6.99 - 2.8) \times 80 \times 1000 \times 10^{-3}$
$= 335.2$ kN/m

Effective prestress $f_{pe} = \eta f_{pi}$
$= 0.75 \times (0.75 \times 1860)$
$= 1046$ N/mm^2

Hence $A_{ps} = P_e / f_{pe}$
$= 335.2 \times 10^3/1046$
$= 320$ mm^2/m

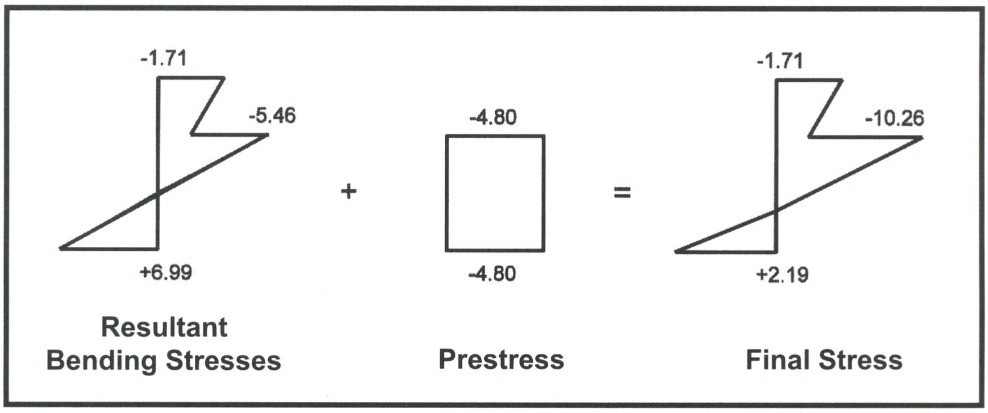
Use ϕ9.6 strands at 150 c/c ($A_{ps} = 367$ mm^2/m)

Actual $P_e = 367 \times 1046 \times 10^{-3}$
$= 383.9$ kN/m

Axial concrete stress in plank $f_{cp} = 383.9 \times 10^3/(80 \times 1000)$
$= 4.80$ N/mm^2

Step 5 : Resultant concrete stresses

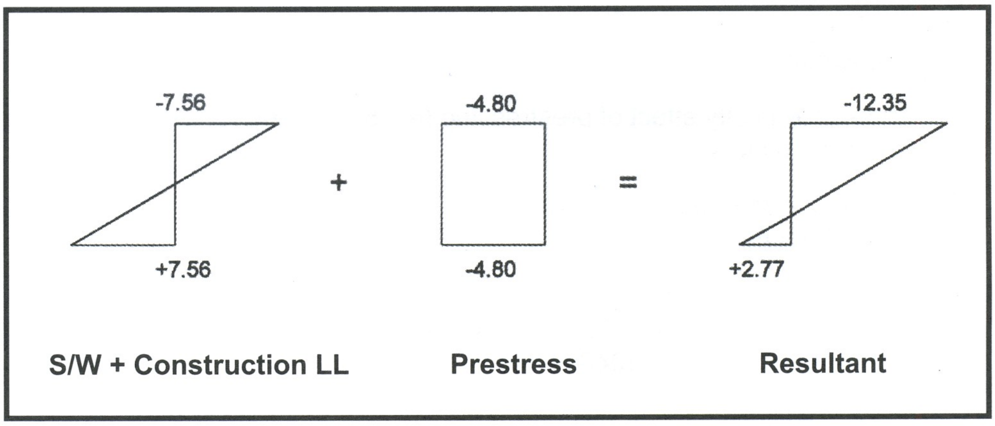
At mid span :



Maximum concrete tension at plank soffit
$= 2.19$ N/mm^2 $< 0.45\sqrt{40}$
$= 2.8$ N/mm^2 OK

Maximum concrete compression at interface
$= 10.26$ N/mm^2 $< 0.33 f_{cu}$
$= 0.33 \times 40$
$= 13.2$ N/mm^2 OK

Step 6 : Check mid-span concrete stresses during installation (assume taking place immediately after transfer)



Maximum concrete tension = 2.77 < 2.8 N/mm² OK

Maximum concrete compression = 12.35 < $0.50f_{ci}$
= 0.50 x 25
= 12.5 N/mm² OK

Step 7 : Deflection

a. At installation

Total service load (including construction live load)

q = 3.48 + 1.50
= 4.98 kN/m²

Deflection $\delta = 5ql^4/(384E_cI)$

l = 3600mm
E_c = 28 kN/mm²
I = $bh^3/12$

$$\delta = \frac{5 \times 4.98 \times 3600^4 \times 12}{384 \times 28 \times 10^3 \times 1000 \times 80^3}$$

= 9.1 mm

δ/l = 1/395 < 1/250 (c1.3.2.1.1, Part 2) OK

b. At service

Total service load (imposed dead load + live load)

q = 1.70 + 2.0
= 3.70 kN/m²

$$\delta = \frac{5 \times 3.70 \times 3600^4 \times 12}{384 \times 28 \times 10^3 \times 1000 \times 145^3}$$

= 1.1 mm
= 1/3272 < 1/350 (c1.3.2.1.2, Part 2) OK

B. Ultimate Limit State Design

Step 8 : Design for bending

a. At installation

Ultimate UDL = 1.4 x 3.48 +1.6 x 1.50
= 7.30 kN/m^2

Ultimate bending moment M_u = 7.30 x $3.6^2/8$
= 11.78 kNm/m

From Part 1, clause 3.4.4.4

$\chi/d = 1.11\,(1 - \sqrt{1 - 4.44M_u / bd^2f_{cu}})$

d = 80 - 35
= 45 mm

b = 1000 mm

$\chi/d = 1.11\,[\,1 - \sqrt{1 - (4.44 \times 11.78 \times 10^6/(1000 \times 45^2 \times 40))}\,]$
= 0.449

χ = 20.2 mm < d/2 = 22.5 mm

Concrete compression, $F_c = 0.45f_{cu}\, b\, \chi$
= 0.45 x 40 x 1000 x 20.2 x 10^{-3}
= 363.6 kN/m

b. At final stage

Ultimate UDL = 1.4 x 1.70 + 1.6 x 2.0
= 5.58 kN/m^2

Ultimate bending moment M_u = 5.58 x $3.6^2/8$
= 9.04 kNm/m

d = 145 - 35
= 110 mm

$\chi/d = 1.11\,[\,1 - \sqrt{1 - (4.44 \times 9.04 \times 10^6/(1000 \times 110^2 \times 35))}\,]$
= 0.0539

χ = 5.9 mm

Concrete compression = $0.45f_{cu}\, b\, \chi$
= 0.45 x 35 x 1000 x 5.9 x 10^{-3}
= 92.9 kN/m

$f_{pu}A_{ps} / f_{cu}bd$ = 1860 x 367 / (35 x 1000 x110)
= 0.18

f_{pe} / f_{pu} = 1046 / 1860
= 0.56

Part 1,Table 4.4, $f_{pb} = 0.87f_{pu} \times 0.95$
= 0.83 x 1860
= 1537 N/mm^2

Total tension to be provided by prestressing tendons,

P_s = 1537 x 367 x 10^{-3}
= 564.1 kN/m > 456.5 kN/m (=363.6 + 92.9) OK

Step 9 : Design for composite action (refer to Section 2.3.4)

Total horizontal force = 456.9 kN/m

Contact width b_e = 1000 mm
Contact length l_e = 3600/2
= 1800 mm

Average v_h = $456.9 \times 10^3/(1000 \times 1800)$
= 0.25 N/mm² OK

For a triangular distribution of shear force, the maximum horizontal shear stress at the support is

v_{hmax} = 0.25×2
= 0.50 N/mm^2

which is less than 0.60 N/mm^2 in Table 5.5, Part 1 of the Code. Hence shear friction reinforcement is not necessary to ensure composite behaviour.

Step 10 : Design for vertical shear

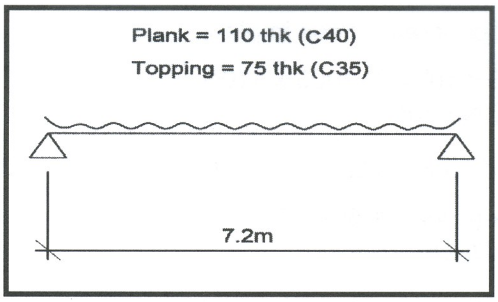
Total vertical shear at support

$V = [1.4(3.48 + 1.7) + 1.6 \times 2.0] \times 3.6/2$
$= 18.8$ kN/m

$v = 18.8 \times 10^3/(1000 \times 110)$
$= 0.17\ N/mm^2 < 0.35\ N/mm^2$ (minimum) OK

Design Example 2 : Propped Prestressed Precast Concrete Planks

Design a 110mm thick prestressed pretensioned plank simply supported at 7.2 m and acting compositely with 75 mm thick concrete topping. All other design data are to be as per Design Example 1. Design live load is 2.5 kN/m^2 and the planks are to be propped at mid-span during installation.

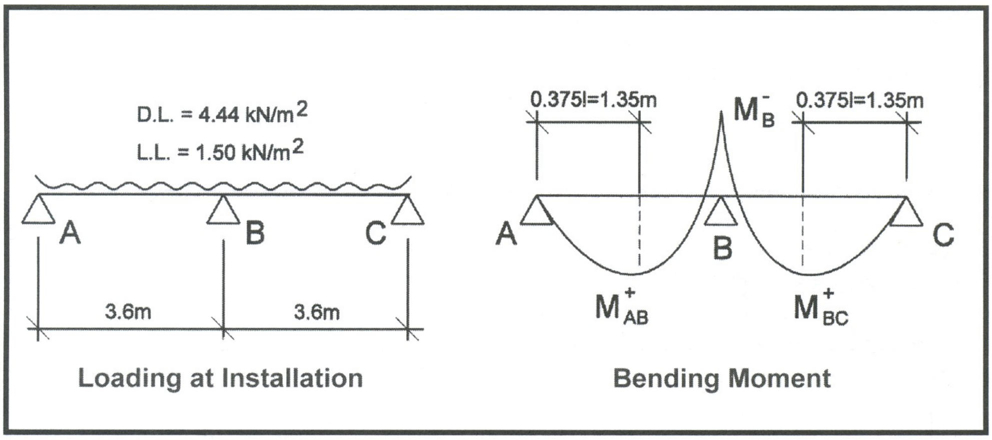


A. Service Stress Design

Step 1 : Calculate bending stress at installation stage.

Loading :

Dead load : plank self weight	= 0.110 x 24	= 2.64 kN/m^2
topping	= 0.075 x 24	= 1.80 kN/m^2
		4.44 kN/m^2
Live load (construction)		= 1.50 kN/m^2



Support :

$M_B^- = 0.125ql^2$

a. Due to DL only : $M_B^- = 0.125 \times 4.44 \times 3.6^2$
$= 7.19$ kNm/m

Top and soffit concrete stress in planks,
$f_c = \pm 7.19 \times 10^6 \times 6/(1000 \times 110^2)$
$= \pm 3.57$ N/mm^2

b. Due to DL and construction live load :
$M_B^- = 0.125 \times (4.44 + 1.50) \times 3.6^2$
$= 9.62$ kNm/m

Top and soffit concrete stress in planks,
$f_c = \pm 9.62 \times 10^6 \times 6/(1000 \times 110^2)$
$= \pm 4.77$ N/mm^2

Span :

M_{AB}^{+} = M_{BC}^{+} = 0.07ql^2 at 1.35 m from end support

a. Due to DL only : M_{AB}^{+} = 0.07 x 4.44 x 3.6^2
= 4.03 kNm/m

Top and soffit concrete stress in planks,

f_c = ± 4.03 x 10^6 x 6/(1000 x 110^2)
= ± 2.00 N/mm^2

b. Due to DL and construction live load :

M_{AB}^{+} = 0.07 x (4.44 + 1.50) x 3.6^2
= 5.39 kNm/m

Top and soffit concrete stress in planks,

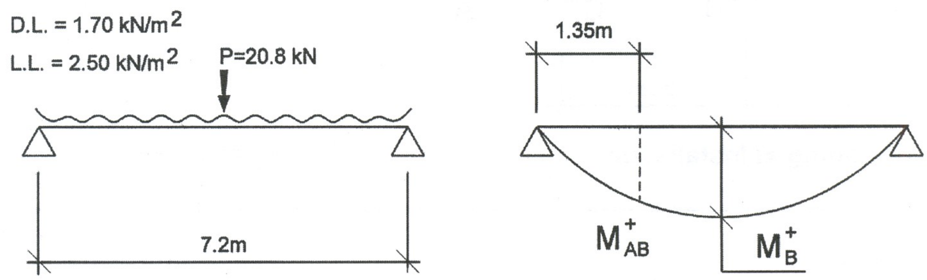
f_c = ± 5.39 x 10^6 x 6/(1000 x 110^2)
= ± 2.67 N/mm^2

Step 2 : Calculate bending stress at final stage

Loading

Dead load :	Finishes	= 1.20 kN/m^2
	Services	= 0.50 kN/m^2
		1.70 kN/m^2
Live load		= 2.50 kN/m^2
Total		= 4.20 kN/m^2

Prop reaction at point B, P = 1.30 ql
= 1.30 x 4.44 x 3.6
= 20.8 kN/m



Loading at Final Stage **Bending Moment**

At mid-span :

$M_B^+ = (4.20 \times 7.2^2/8) + (20.8 \times 7.2/4)$
$= 64.66$ kNm/m

Top and soffit concrete stress in composite section,

$f_c = \pm 64.66 \times 10^6 \times 6/(1000 \times 185^2)$
$= \pm 11.34$ N/mm^2

At 1.35 m from support :

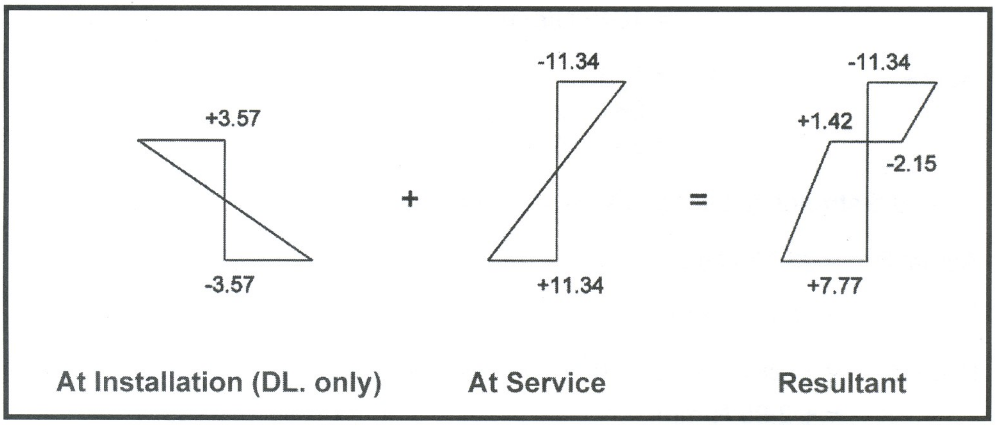
$M_{AB}^+ = (10.4 + 15.12) \times 1.35 - 4.20 \times 1.35^2/2$
$= 30.62$ kNm/m

Top and soffit concrete stress in composite section,

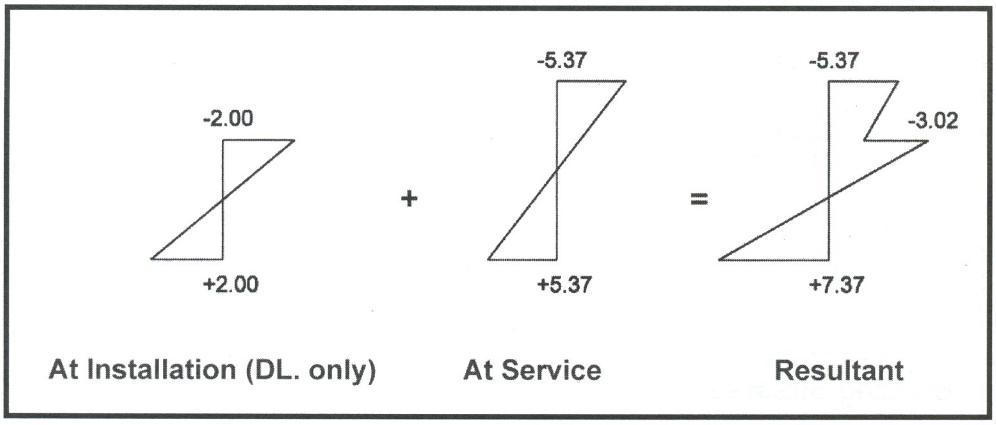
$f_c = \pm 30.62 \times 10^6 \times 6/(1000 \times 185^2)$
$= \pm 5.37$ N/mm^2

Step 3 : Resultant bending stress

a. At mid span :



b. At 1.35m from support :



Maximum concrete tensile stress is at mid-span at 7.77N/mm^2

Step 4 : Calculate effective prestressing force and reinforcement

Permissible tensile stress at plank soffit (class 2)

$f_t = 0.45\sqrt{f_{cu}}$

$= 2.8\ N/mm^2$

Eccentricity of prestressing force to plank centroid

$e = (110/2) - 35$

$= 20\ mm$

Concrete stress at the soffit of plank at mid-span,

$7.77 - P_e / A_c - P_e / Z_b = 2.8$

$7.77 - P_e / (1000 \times 110) - P_e \times 20 \times 6 / (1000 \times 110^2) = 2.8$

$P_e = 261.5\ kN/m$

Effective prestress $f_{pe} = \eta f_{pi}$

$= 0.75 \times (0.75 \times 1860)$

$= 1046\ N/mm^2$

Hence $A_{ps} = P_e/f_{pe}$

$= 261.5 \times 10^3/1046$

$= 250\ mm^2/m$

Use ϕ9.6 strands at 200 c/c ($A_{ps} = 275\ mm^2/m$)

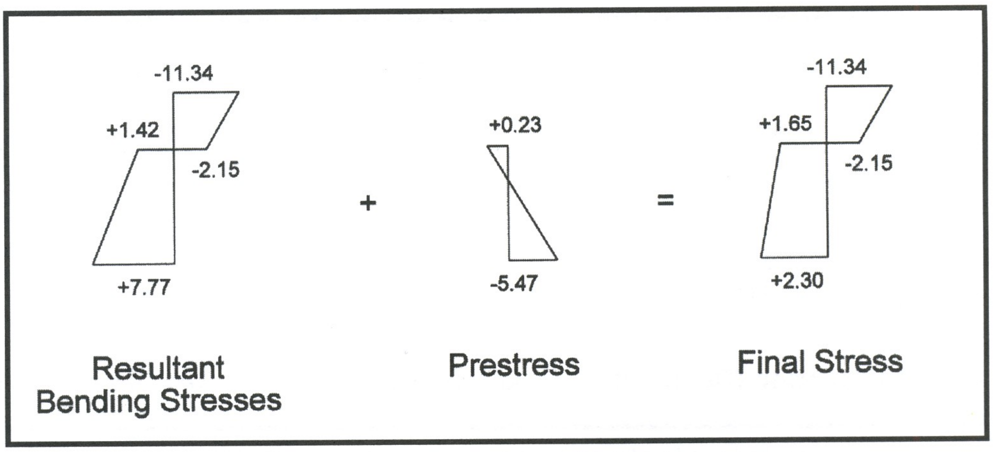
Actual P_e = 287.7 kN/m

P_e/A_c = -2.62 N/mm²

$P_e e / Z_b = \pm (287.7 \times 20 \times 10^3 \times 6)/(1000 \times 110^2)$

$= \pm 2.85\ N/mm^2$

Step 5 : Resultant final concrete stresses



Stresses At Mid-Span

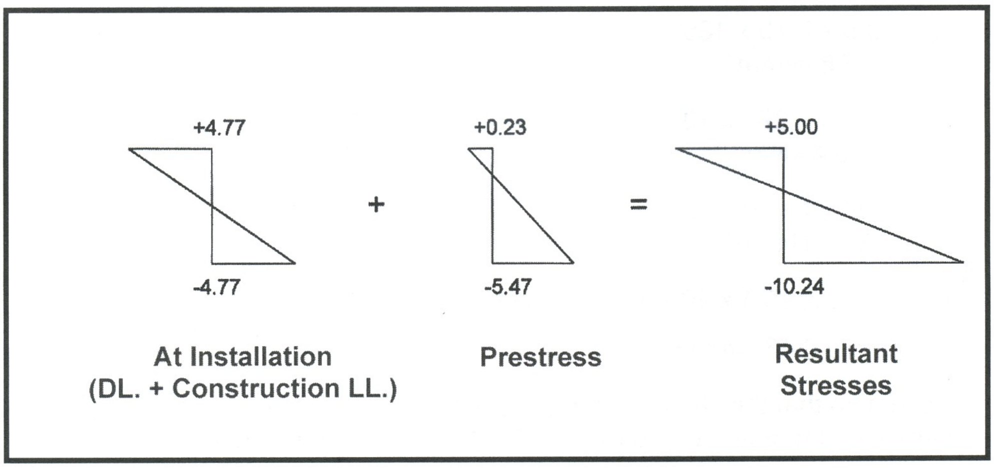
Maximum tensile stress $= 2.30 < 2.80\ N/mm^2$ OK

Maximum compression $= 11.34 < 0.33\ f_{cu}$ OK

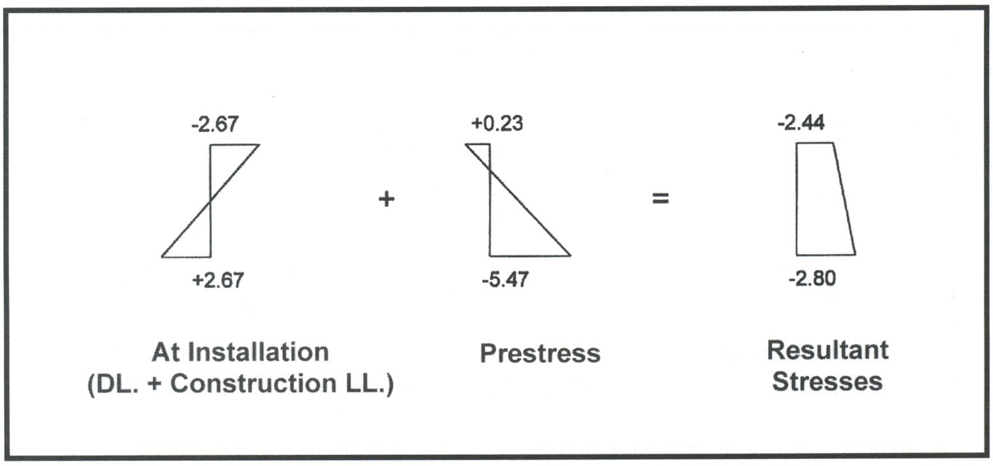
$= 11.5\ N/mm^2$

Step 6 : Check stresses at installation

a. At Propping Support :



b. At 1.35m From End Support :



Maximum stresses are at the propping point.
Maximum compression = 10.24 N/mm^2 < 0.5 f_{ci} (0.5 x 25 = 12.5N/mm^2) OK

Note: Maximum tension found at the top face of the plank is + 5.00 N/mm^2. This is within the class 3 hypothetical tensile stress of 5.5N/mm^2 (0.2 mm crack width) for C40 concrete. The top section will eventually be under permanent compression as the interface is above the neutral axis of the composite action. Another point to note is that the construction live load is transient and when it is removed, the actual tension at the top face over the propping support is (3.57 + 0.23) = 3.8 N/mm^2, which is well within class 3 stresses with 0.1 mm crack width. Cracks at the top face over the prop, if any, will not affect the structural integrity of the planks.

Step 7 : Check concrete stresses at transfer

Assume prestress loss ratio 0.9 at transfer

f_{pi} = 0.9 x 0.75 x 1860
= 1256 N/mm^2

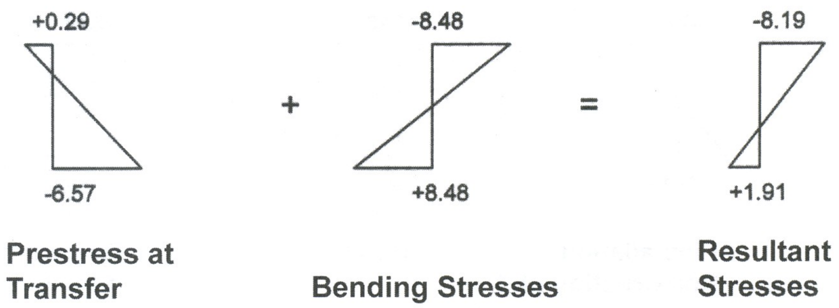
P_i = 1256 x 275 x 10^{-3}
= 345.4 kN/m

f_{ci} = -345.4 x 10^3/(1000 x 110)
= -3.14 N/mm^2

$P_i\, e/Z_b$ = ± (345.4 x 20 x 10^3 x 6)/(1000 x 110^2)
= ± 3.43 N/mm^2

Assuming that the plank is simply supported after transfer, the bending stresses due to a self-weight moment of 2.64 x 7.2^2/8 = 17.11 kNm/m at mid-span is

f_c = ± (17.11 x 10^6 x6)/(1000 x 110^2)
= ± 8.48 N/mm^2



Maximum compression = -8.19 N/mm^2
Maximum concrete compressive strength required at transfer
f_{ci} = 8.19/0.5
= 16.4 N/mm^2 < 25 N/mm^2 OK

Step 8 : Deflection

Deflection at installation is not critical as the planks are centrally propped.

Deflection at final stage :

δ = 5ql ^{4} / 384E_cI + Pl^3 / 48E_cI
q = 1.70 + 2.50 = 4.20 kN/m^2
P = 20.8 kN/m
E_c = 28 kN/mm^2
I = bh^3/12
l = 7200mm

δ = 5 x 4.20 x 7200^4 x 12 / (384 x 28 x 10^3 x 1000 x 185^3) +
20.8 x 10^3 x 7200^3 x 12 / (48 x 28 x 10^3 x 1000 x 185^3)
= 9.9 + 10.9
= 20.8 mm

δ/l = 20.8/7200
= 1/346 ≈ 1/350 OK

B. Ultimate Limit State Design

Step 9 : Design for bending moment

Ultimate UDL = 1.40 x 1.70 + 1.60 x 2.50
= 6.38 kN/m^2

Ultimate prop reaction = 1.4 x 20.8
= 29.1 kN/m

At mid-span : M_u = 6.38 x 7.2^2 / 8 + 29.1 x 7.2 / 4
= 93.7 kNm/m

Negative moment at mid-span moment during installation stage (step 1 (a))

M_u^- = 1.4 x 7.19
= 10.1 kNm/m

Hence net mid-span moment
M_u = 93.7 - 10.1
= 83.6 kNm/m

χ/d = $1.11[1 - \sqrt{1 - (4.44M_u/bd^2f_{cu})}]$
d = 185 - 35
= 150 mm

χ/d = $1.11[1 - \sqrt{1 - (4.44 \times 83.6 \times 10^6/(1000 \times 150^2 \times 35))}]$
= 0.303
χ = 45.4 mm

Concrete compression = $0.45f_{cu}\, b\, \chi$
= 0.45 x 35 x 1000 x 45.4 x 10^{-3}
= 715.0 kN/m

Total tension to be provided by prestressing tendons

= $0.87f_{pu}A_{ps}$ x 1.0 (From Part 1, Table 4.4)
= 0.87 x 1860 x 275 x 10^{-3} x 1.0
= 445.0 kN/m

Additional tension capacity to be provided by normal reinforcement

A_s = (715 - 445) x 10^3/(0.87 x 460)
= 675 mm^2/m

Use T10 @ 100 c/c (A_s = 785 mm^2/m), placed at the same level with the tendons.

Step 10 : Design for composite action

Total horizontal force = 715.0 kN/m
Contact width, b_e = 1000 mm
Contact length, l_e = 7200/2
= 3600 mm
Average v_h = 715.0 x 10^3/(1000 x 3600)
= 0.20 N/mm^2 OK

Proportioning to shear force distribution maximum horizontal shear stress at support

v_{hmax} = 2 x 0.20
= 0.40 N/mm^2

which is less than 0.6 N/mm^2 (Table 5.5, Part 1) OK

Step 11 : Design for vertical shear

Total vertical shear at support

$V = 6.38 \times 3.6 + 29.1/2 + 1.4 \times 4.44 \times 0.375 \times 3.6$
$= 45.9$ kN/m

$v = 45.9 \times 10^3/(1000 \times 150)$
$= 0.30$ N/mm^2 < min 0.35 N/mm^2 OK

2.3 Design Of Precast Reinforced Concrete Beams

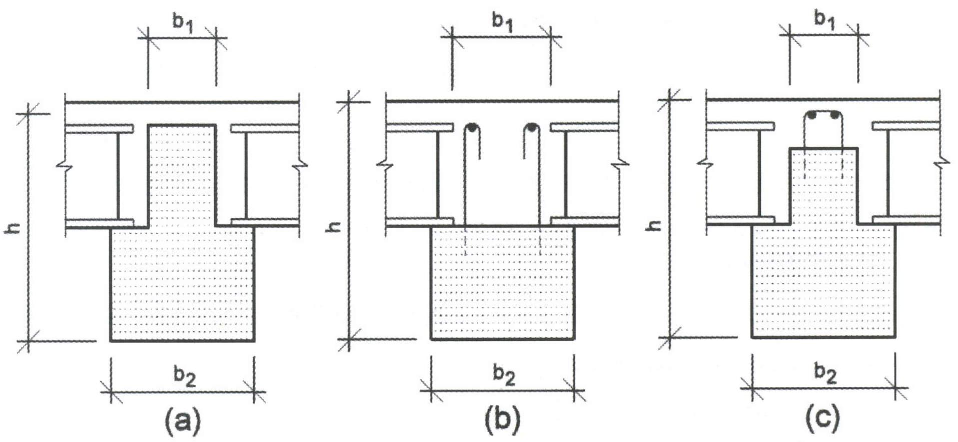
2.3.1 Design considerations

The design of precast reinforced concrete beams is affected by the following factors :

1. section properties of the precast beam,
2. construction methods,
3. sequence of the loads applied onto the beams, and
4. beam behaviour at the serviceability and ultimate limit state

2.3.2 Beam sections

Precast beams may be designed in either full, semi-precast or shell sections depending on the fabrication, jointing details, handling, delivery and lifting capacities of the cranes. The widths and depths in Figure 2.16 may be used in the design of the beam sections:



Flexural bending reinforcement use	$\approx b_1$ at span
	$\approx b_2$ at support (for continuous beam)
Shear reinforcement	$\approx b_1$
Composite shear links	$\approx b_1$ in (b) and (c)

Note : (a) full precast section
(b), (c) semi-precast sections with composite topping concrete

Figure 2.16 Effective Widths And Depths In Precast Beams Design

2.3.3 Construction methods and loading considerations

Figure 2.17 illustrates the various methods in construction using precast beams which can be broadly grouped into propped or unpropped construction with full or semi-precast sections. The final beam behaviour can be either simply supported or with semi-rigid or rigid moment connection at the supports for continuous composite beam behaviour.

At the installation stage, the load consists of essentially the self-weight of the beam, floor elements and wet concrete topping. In unpropped construction, the loads are carried wholly by the precast beams whereas in propped construction, part or all of the loads will be transferred to the props.

On removing the props, additional moments and shears will be created by the prop reactions which will be carried by the composite action of the beams. Precise instructions must, therefore, be given on the method of construction of the precast beams and the positions of the props if they are required.

At the service stage, the stresses in the beams are primarily due to imposed dead and live loads. Depending on the construction methods, the loading considerations on the beam design can be categorised into the following cases:

1. **unpropped construction with simply-supported beam behaviour or continuously propped precast beam :**

   The loads are applied as in the conventional cast in-situ beams design.

2. **unpropped construction with full or semi-precast section with continuous beam behaviour :**

   Apart from the dead and live loads, the beams are subjected to an additional live load of:

   0.4/1.6 x (beam self-weight + floor element + wet concrete)

   The load is treated as live load and is applied in order to satisfy the critical loading arrangement required in Part 1, clause 3.2.1.2.2, of the Code.

3. **propped construction with semi-precast section and continuous beam behaviour :**

   In addition to the imposed dead and live loads and the equivalent live load in (2) above, the beams will also be subjected to the action of prop forces. These are applied as point loads acting vertically downwards at the respective position of the props.

   The loading consideration of the semi-precast continuous beams in (2) and (3) above is illustrated in Figure 2.18.

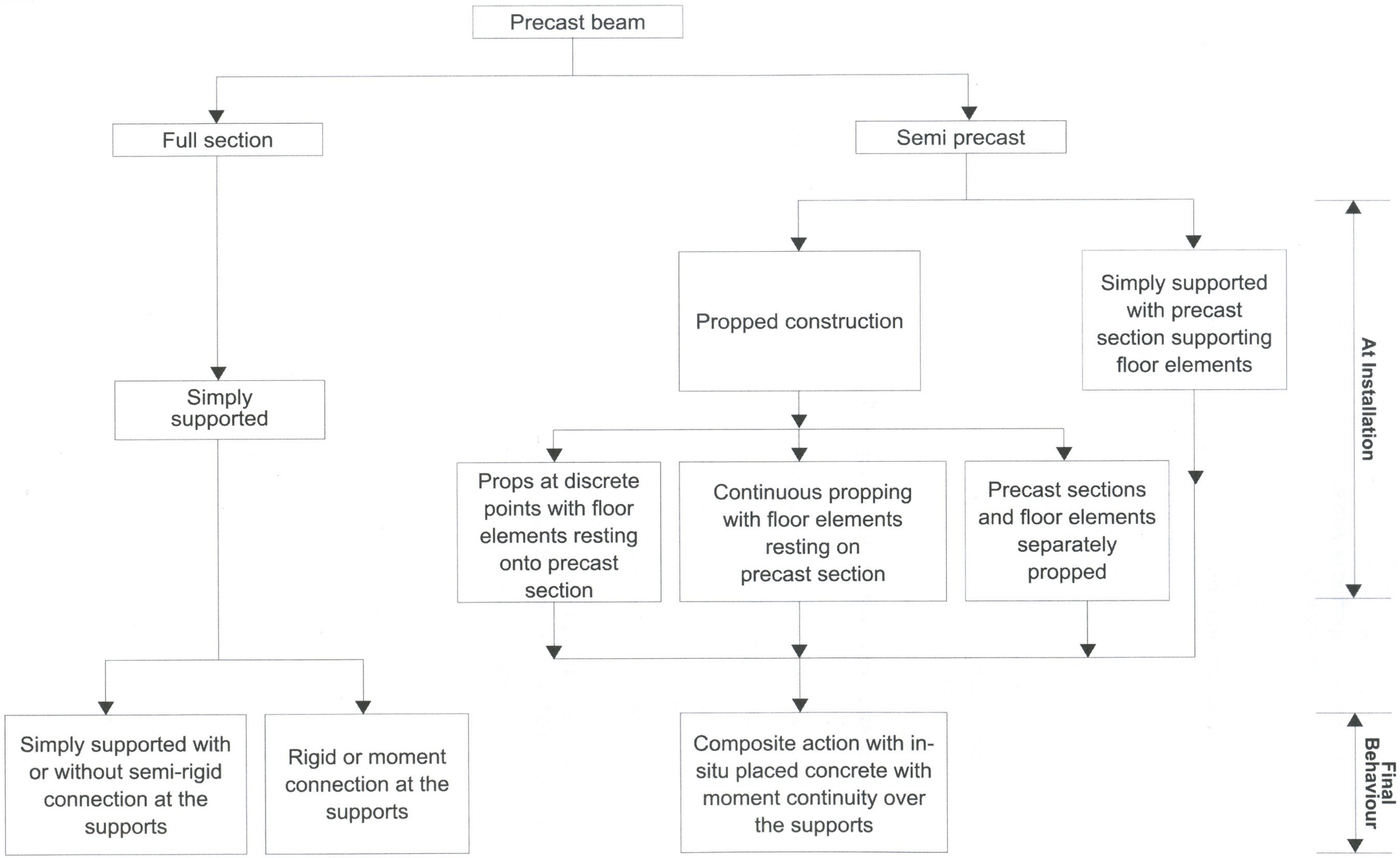
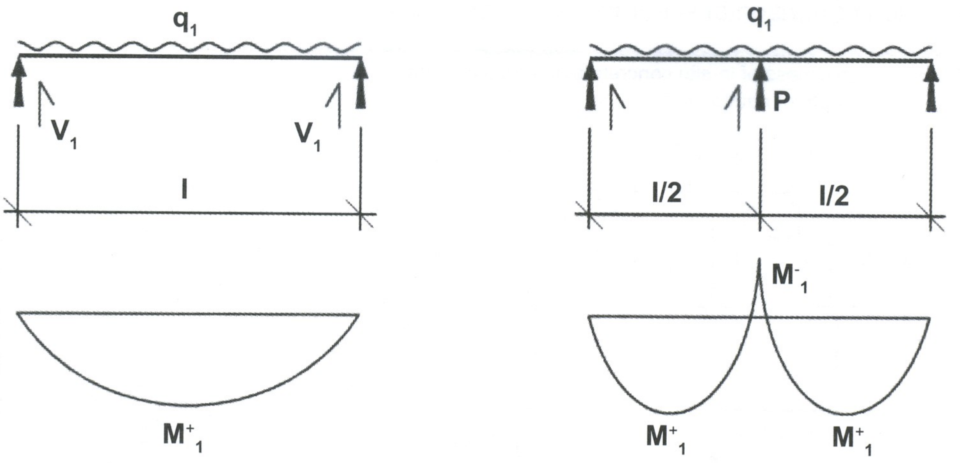


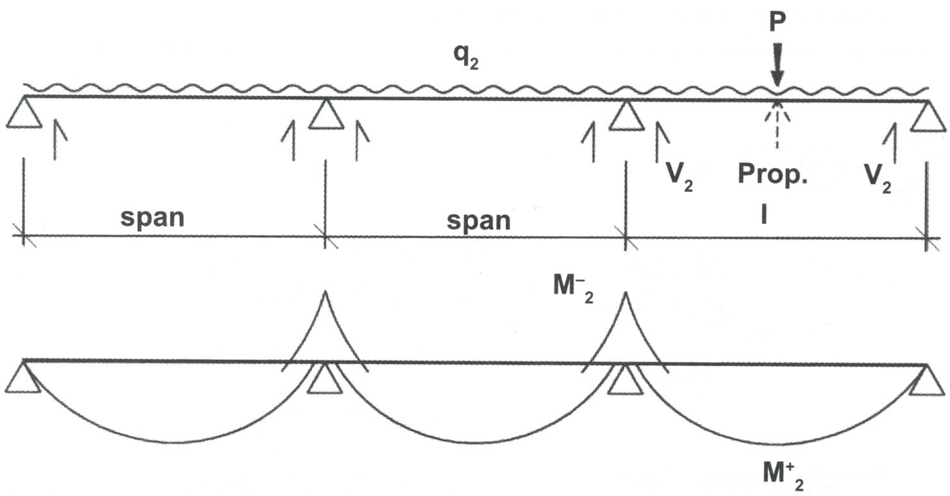
Figure 2.17 Precast Concrete Construction Methods And Beam Behaviours



Unpropped Construction **Propped Construction**

q_1 = Weight of beam + floor elements + wet concrete topping (load factor = 1.0)

(a) Semi-Precast Beams Supporting Floor Elements



q_2 = 1.4 x imposed dead load + 1.6 x (imposed live load + 0.4/1.6 x q_1)
P = reverse prop reaction (load factor = 1.4)

(b) Continuous Composite Beams At Ultimate Limit State

The final design moment is given as

Support moment = M_2^-
Span moment = $1.0M_1^+ + M_2^+$

The design shear force at support is given as

Total shear $V = 1.0V_1 + V_2$

Figure 2.18 Loading Consideration In Semi-Precast Continuous Beam Design

2.3.4 Design for composite action

Design for composite action may follow the procedures under Part 1, clause 5.4.7, of the Code.

The determination of horizontal shear forces in composite design is shown in Figure 2.19 below :

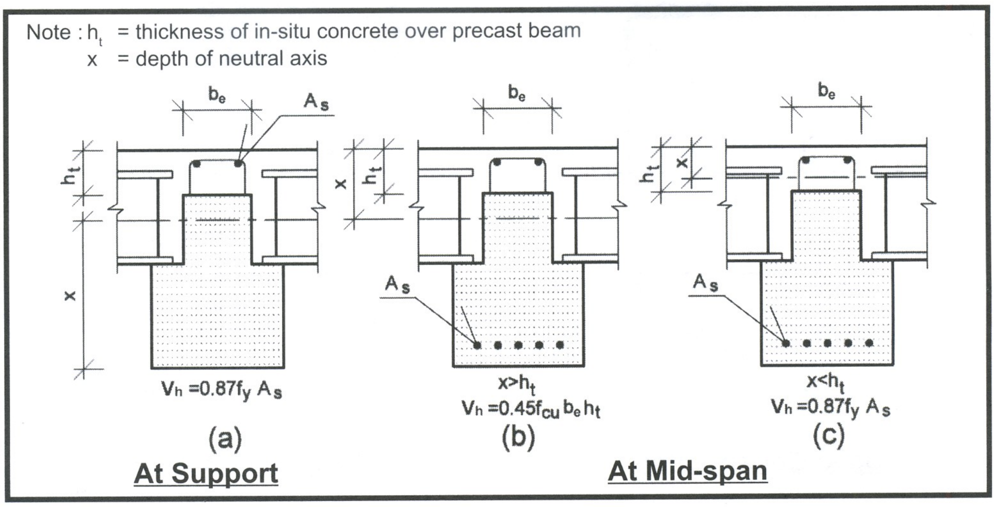


Figure 2.19 Horizontal Shear Force In Composite Concrete Section

The effective contact lengths may be determined as shown in Figure 2.20 :

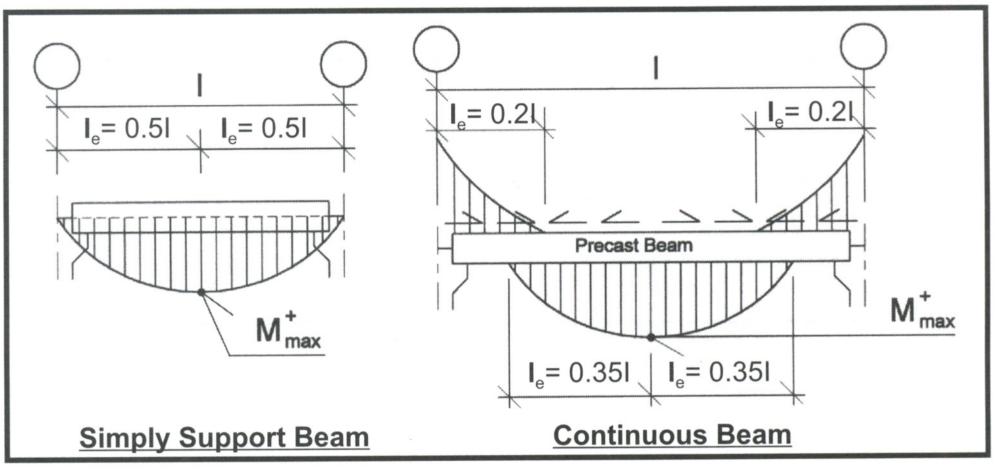


Figure 2.20 Effective Contact Lengths In Composite Design

The required shear links for composite action are calculated from :

$$\frac{A_{sv}}{s_v} = \frac{b_e v_h}{0.87 f_{yv}}$$

where
- b_e = effective contact width
- A_{sv} = area of shear links
- s_v = spacing of shear links
- v_h = average horizontal shear stress $= V_h/(b_e l_e)$

If the average horizontal shear stress is less than the permissible values in Table 5.5 of the Code, only norminal links equivalent to 0.15% of the contact area need to be provided. It is to be noted that the provisions of shear links is based on the larger of the requirements for vertical and horizontal shear and not addition of the two values.

2.3.5 Deflection

Deflection under serviceability requirements can be based on the effective span/depth approach in Part 1, clause 3.4.6 of the Code. When deflection needs to be determined, it may be calculated using the method outlined in Part 2, clause 3.7, BS 8110, which is given by:

$$\delta = Kl^2/r_b$$

and $$l/r_b = M_s/E_c I_e$$

where
- K = coefficient determined from Part 2, Table 3.1 of the Code
- l = span of the beam
- l/r_b = the mid-span curvature or, for cantilevers, at the support section
- M_s = bending moment at span or, for cantilevers, at the support section
- I_e = the effective moment of inertia of the beam
- E_c = modulus of elasticity of concrete

The effective moment of inertia of the beam I_e is calculated from :

$$I_e = (M_{cr} / M_s)^3 I_g + [1 - (M_{cr} / M_s)^3] I_{cr}$$

where
- M_{cr} = cracking moment (= $0.67\sqrt{f_{cu}}\, Z_b$)
- M_s = service load moment
- I_g = gross uncracked moment of inertia of the beam
- I_{cr} = cracked moment of inertia of the beam
- f_{cu} = design concrete cube strength
- Z_b = gross uncracked section modulus at tension face

In simply supported beams, I_e is calculated based on the mid-span value. For continuous beam, a weighted average of the support and span is more appropriate due to the varying degree of cracking at these two regions. The weighted average I_e is calculated from :

continuous span $$I_e = 0.70\, I_{em} + 0.15(I_{e1} + I_{e2})$$

continuous span with simply supported at one end $$I_e = 0.85\, I_{em} + 0.15\, I_{e1}$$

where
- I_{em} = effective moment of inertia for the mid-span
- I_{e1}, I_{e2} = effective moment of inertia for the negative moment sections at the beam ends

For long-term deflection calculations, effective modulus of elasticity $E_{ce} = E_c / (1 + \phi)$ is used where ϕ is the creep coefficient. In general, E_{ce} may be taken to be equal to about $0.5E_c$ unless a more precise ϕ has to be determined.

2.3.6. Crack width

Calculation of crack widths can follow the procedures in Part 2 clause 3.8, of the Code. The calculation of crack widths for beams with concrete covers under normal exposure conditions may not be necessary if the bar spacing rules in Part 1 clause 3.12.11, are observed. The permissible crack width is 0.3 mm for normal reinforced concrete section.

2.3.7 Design charts

The design charts for precast reinforced concrete beams are shown in Figures 2.21 to 2.25 for concrete grades of C30 to C50 respectively. In each of the design chart, the load capacity curves for beams with overall depths from 400 mm to 1200 mm at 100 mm increments are shown. The load capacity curves are based on a beam module of 50 mm width with simply supported conditions. The use of 50 mm module as a beam element has the following advantages :

1. For a given beam depth, the charts are able to cater to different beam widths with simple calculations. This reduces drastically the number of design charts.

2. It provides the designer quick means to explore other options in precasting the beams without carrying out extensive calculations.

The design charts are developed based on a main steel content of between 175 to 200 kg/m^3 of concrete. Including shear links and normal lappings, the steel contents in the final beam section may be 200 to 250 kg/m^3, which may be considered an optimum quantity in a beam section.

There are essentially three basic steps in the use of the charts:

Step 1. Calculate the ultimate uniform floor loading (i.e. 1.4DL + 1.6LL). Self weight of the precast beams need not be considered as the load capacity curves have incorporated the self-weight effect.

Step 2. Assume initially a beam width, b, which is in multiples of 50 mm and work out the total number of 50 mm modules for the assumed width i.e. n = b/50. Divide the ultimate uniform floor loads from Step 1 by n to obtain the loads on each beam module.

Step 3. Knowing the loaded span of the floor, the span of the precast beam and the uniform floor load on a 50 mm wide beam module, the designer can obtain the beam depth and hence the beam section from the design charts.

The design aids also include a bending steel design chart in Figure 2.26 and a permissible shear stress chart in Figure 2.27 which are derived from the code provisions in Part 1 clause. 3.4.4 and 3.4.5 respectively.

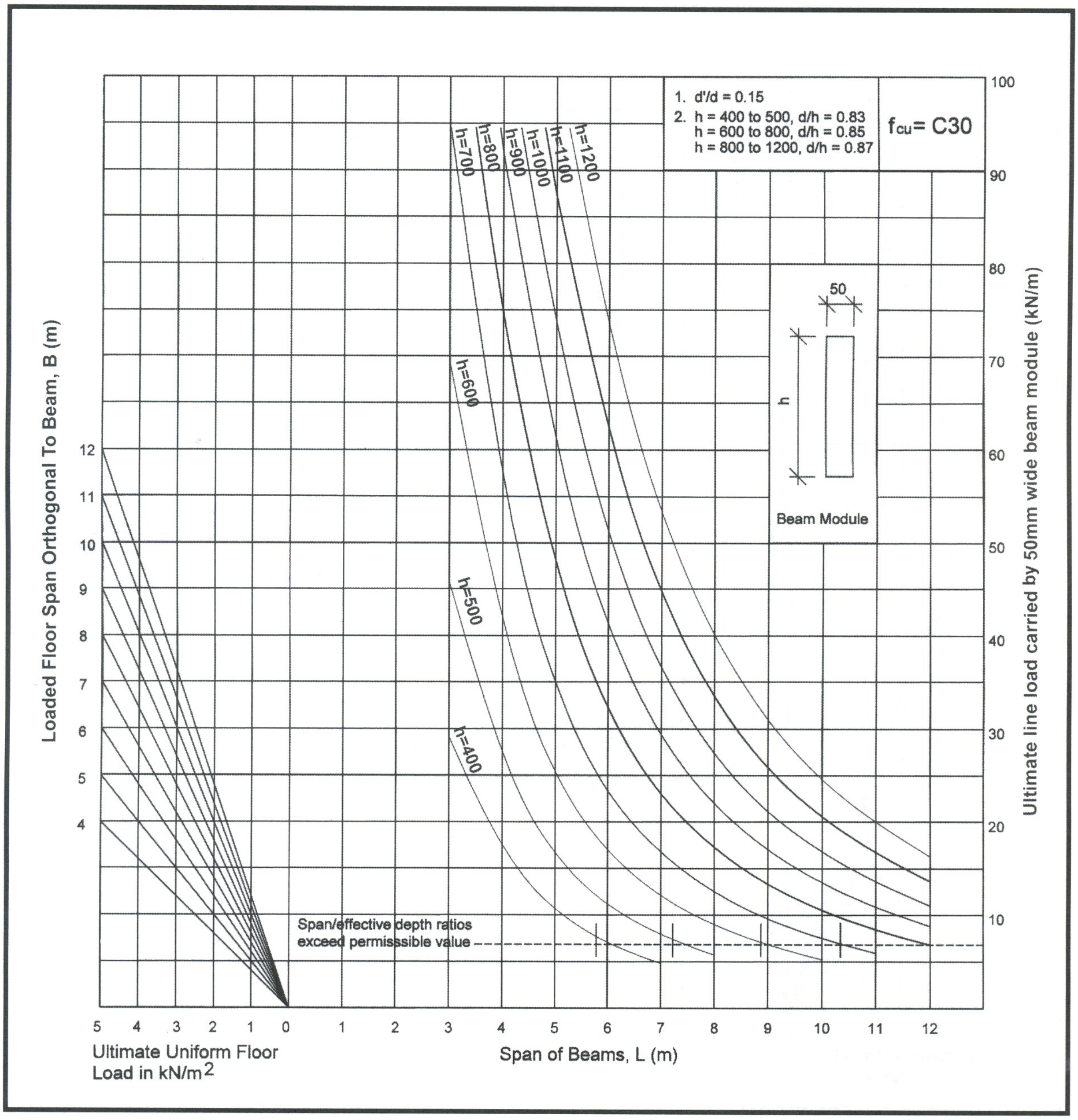


Figure 2.21 Reinforced Concrete Precast Beams Design Chart For f_{cu} = 30N/mm²

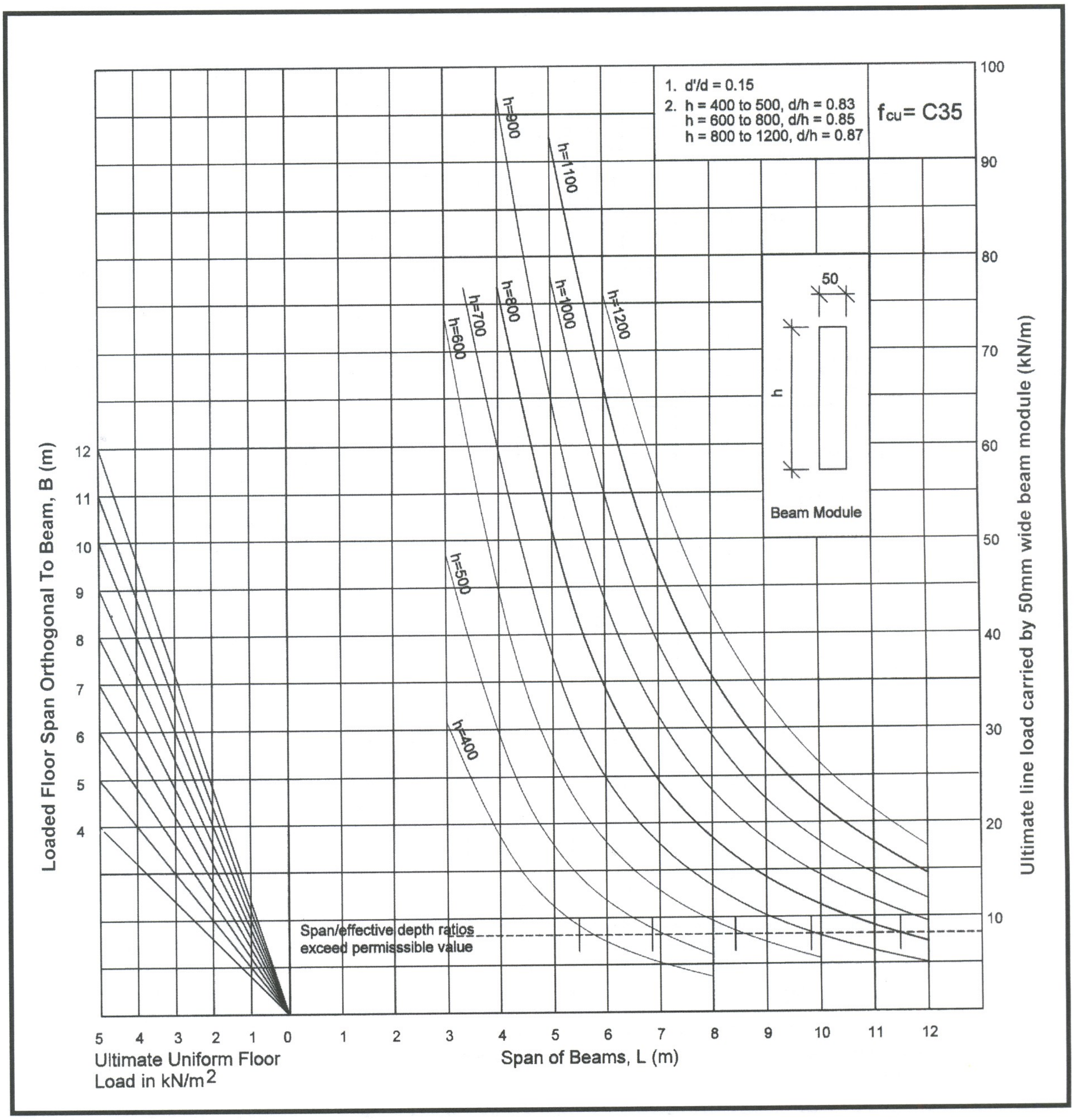


Figure 2.22 Reinforced Concrete Precast Beams Design Chart For f_{cu} = 35N/mm²

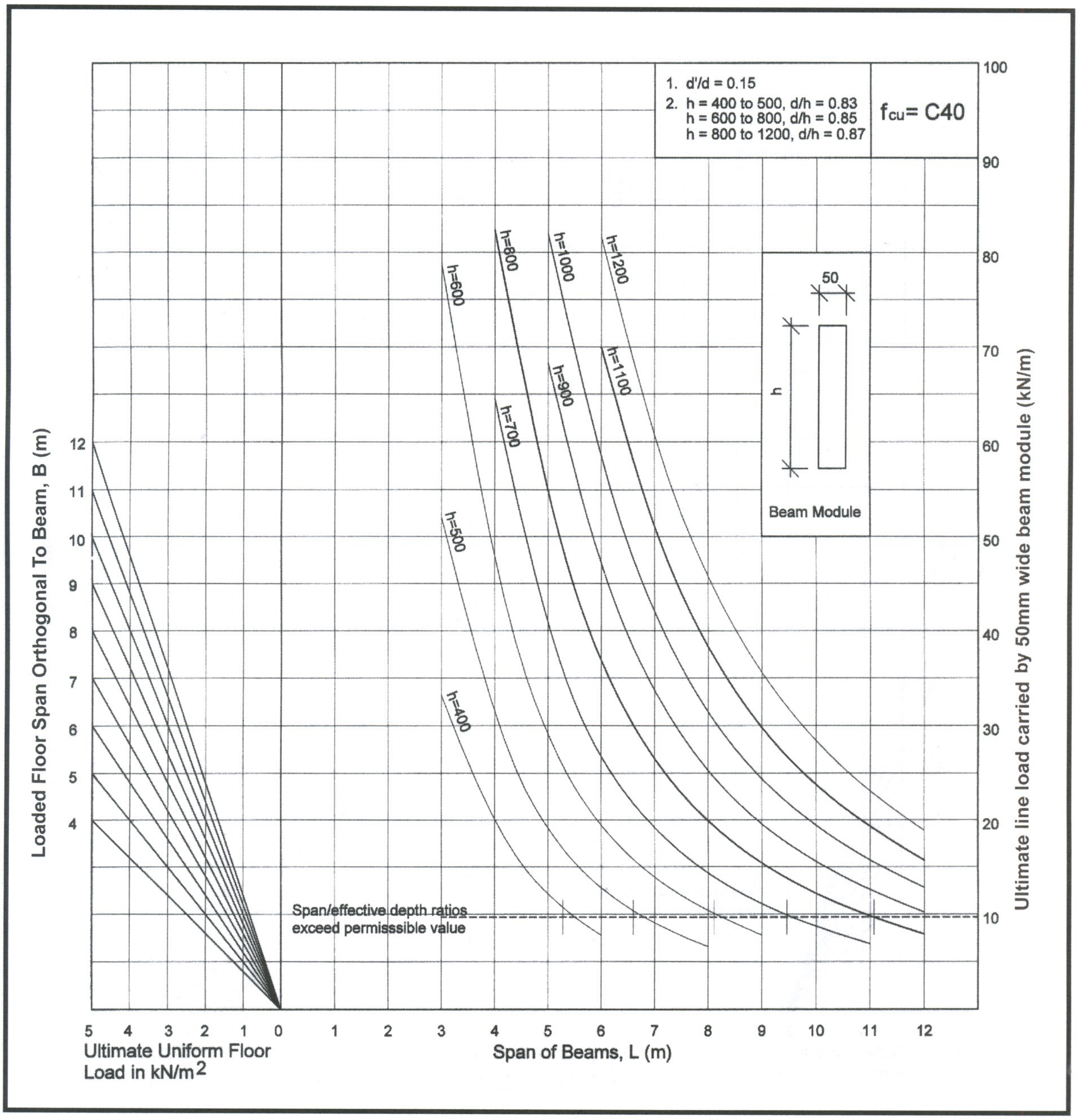


Figure 2.23 Reinforced Concrete Precast Beams Design Chart For f_{cu} = 40N/mm²

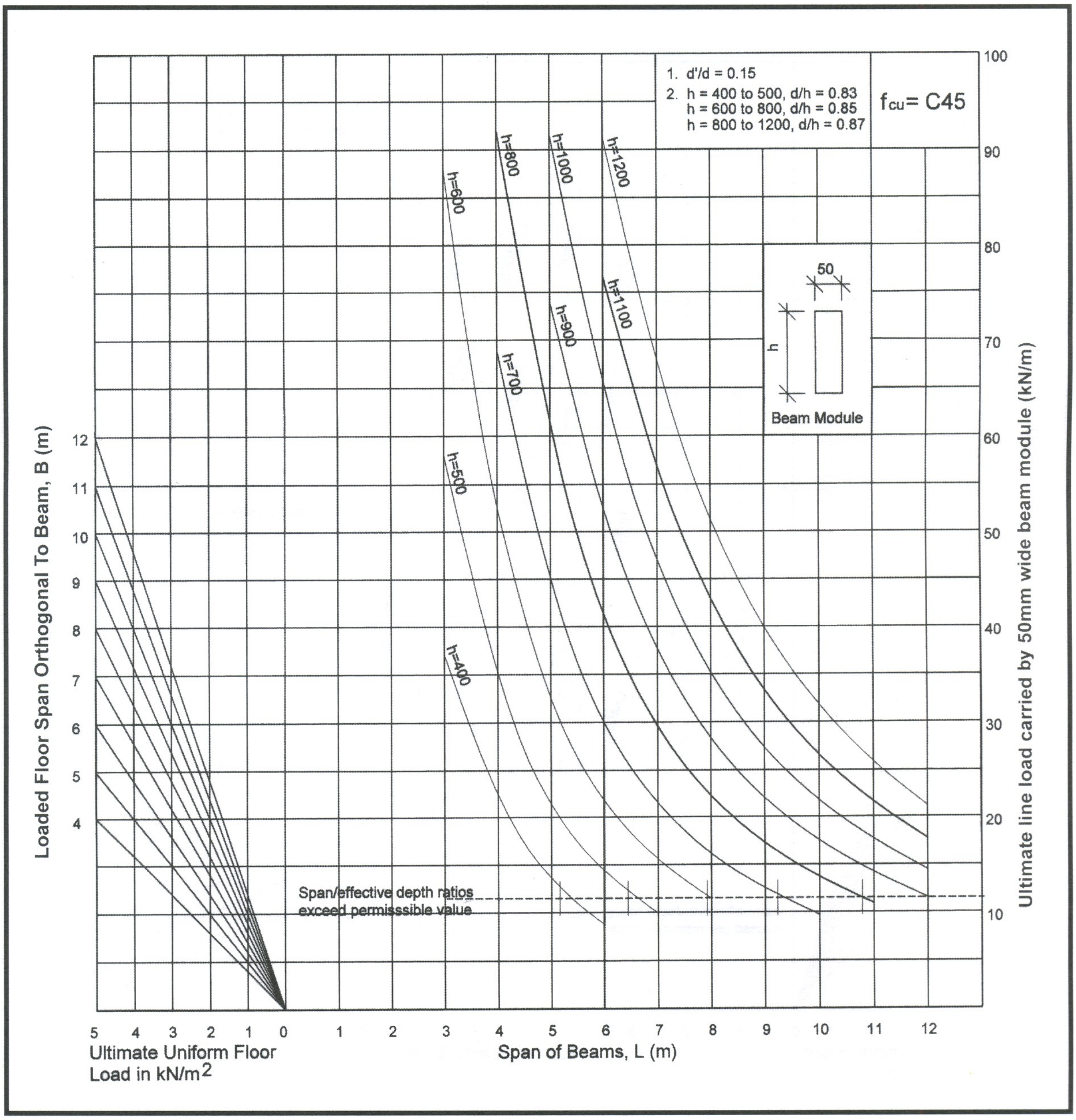


Figure 2.24 Reinforced Concrete Precast Beams Design Chart For f_{cu} = 45N/mm²

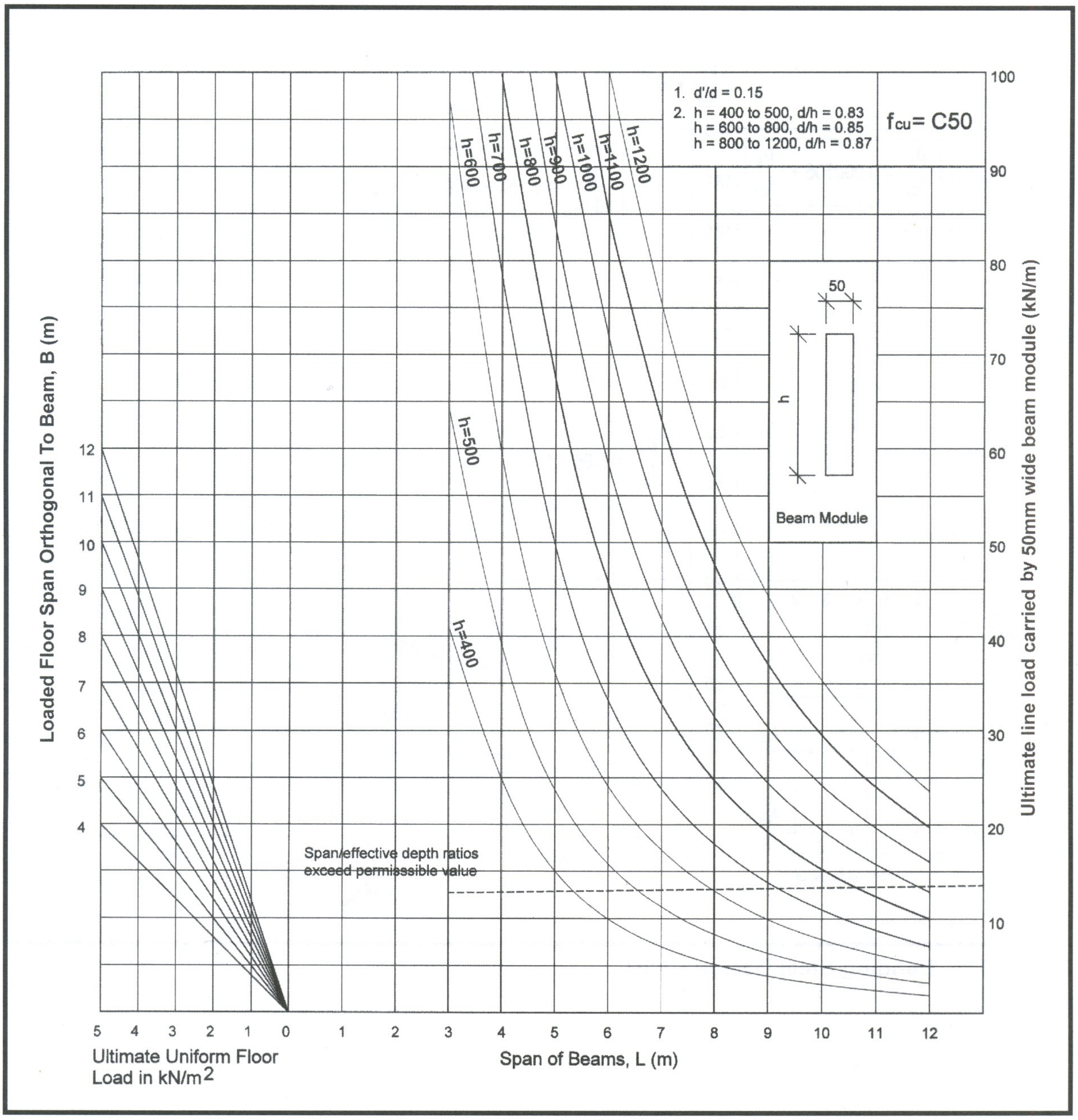


Figure 2.25 Reinforced Concrete Precast Beams Design Chart For f_{cu}=50N/mm^2

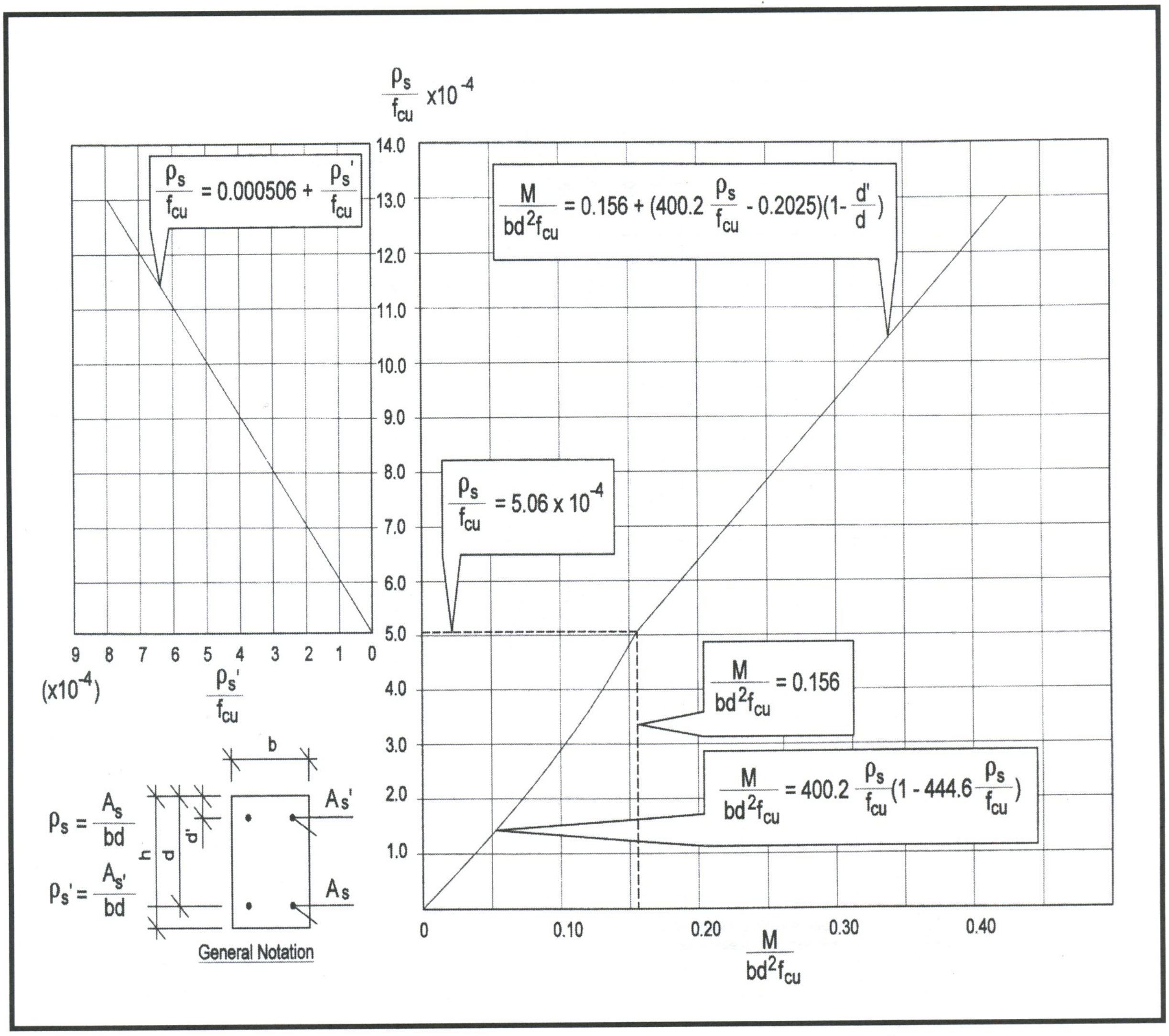


Figure 2.26 Bending Steel Design Chart

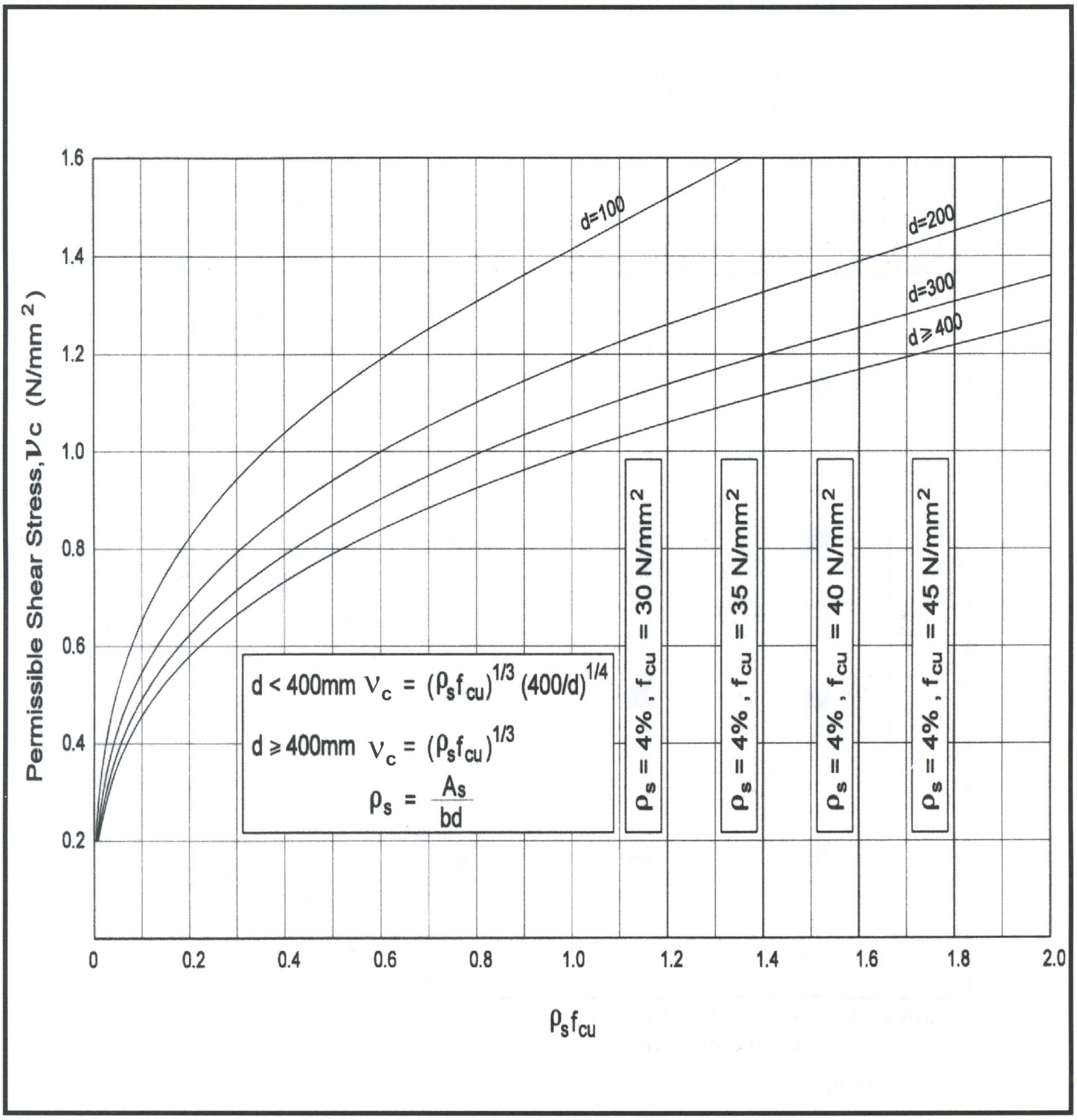


Figure 2.27 Permissible Concrete Shear Stress

Note : The v_c values from the chart will need to be multiplied by a factor 1.063 to give the permissible v_c value in CP65.

2.3.8 Design examples

To illustrate the use of the design charts, four design examples are presented. An internal precast beam, shown in the typical floor plan of an office in Figure 2.28 is used and the variations in the design examples are as follows:

Example 3 : Full precast section, unpropped construction and simply supported final beam behaviour. The overall depth of the beam is restricted by clear headroom requirement.

Example 4 : Resize and design of the precast beam in Example 3 if there is no restriction on the overall beam depth.

Example 5 : Semi-precast section, unpropped construction with final continuous beam behaviour.

Example 6 : Semi-precast section, propped construction with final continuous beam behaviour.

Although Example 6 does not involve the use of any of the charts in Figures 2.21 to 2.25, it is intended to illustrate the use of charts in Figures 2.26 and 2.27 which are applicable to both precast and conventional in-situ beam design.

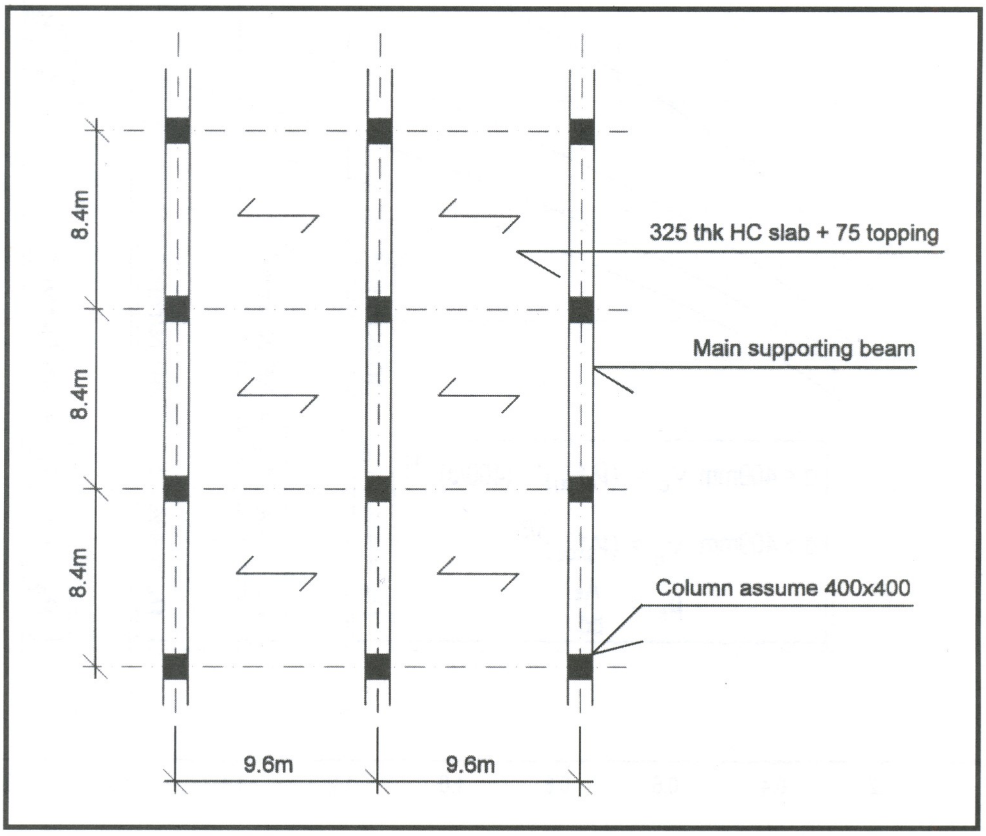


Figure 2.28 Typical Floor Plan For Precast Beam Design For Design Examples 3 to 6.

1. Design Loading :

Imposed dead load	:	Finishes	=	1.20 kN/m^2
		Services	=	0.50 kN/m^2
		Partition	=	1.00 kN/m^2
Imposed live load			=	5.00 kN/m^2

2. Materials

Concrete (C35)	f_{cu}	=	35 N/mm^2
Steel	f_y	=	460 N/mm^2

3. Net span of precast main beam = 8 m

Design Example 3: Full Section Precast Beam And Unpropped Construction

Design the simply supported main precast beams which are to be cast in full section. The beams are to be unpropped during installation. Due to ceiling height requirements, overall depth of the beam should not exceed 700 mm.

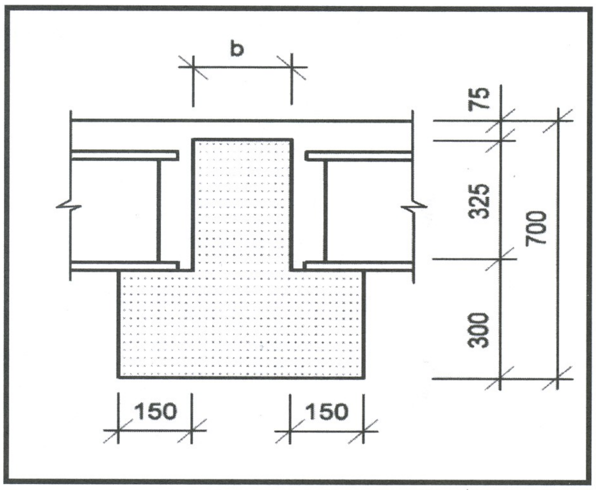
Step 1 : Calculate ultimate floor loading

Dead load:

HC slabs (jointed weight)	=	4.50 kN/m^2
Topping (75 mm thk)	=	1.80 kN/m^2
Finishes	=	1.20 kN/m^2
Services	=	0.50 kN/m^2
Partition	=	1.00 kN/m^2
Total dead load	=	9.00 kN/m^2
Live load	=	5.00 kN/m^2
Ultimate UDL = 1.4DL + 1.6LL	=	20.6 kN/m^2

Step 2 : Determine beam depth and width

Depth, h = 700 - 75
= 625 mm



Typical Beam Section

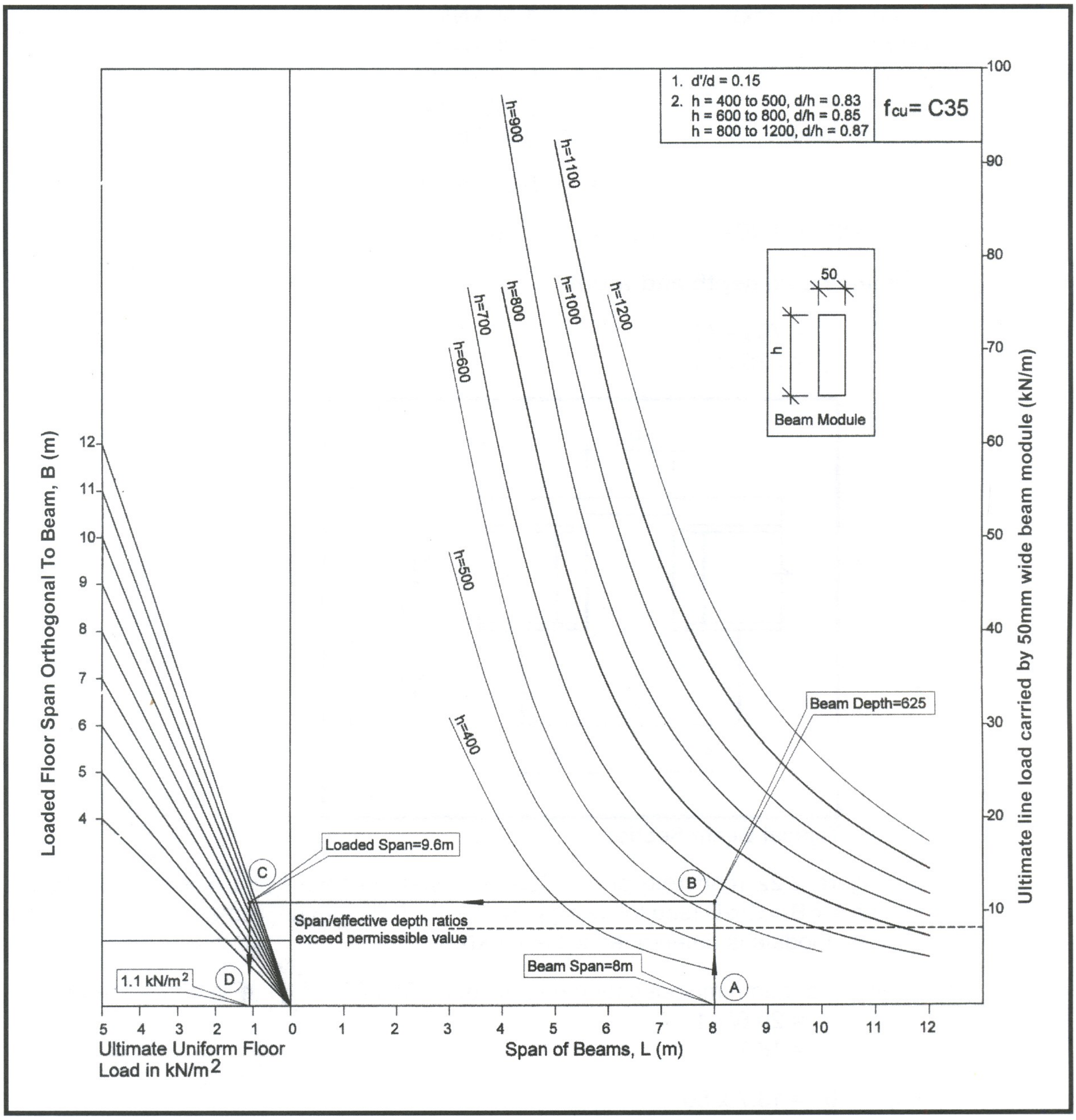
Using Figure 2.22 (concrete grade C35) and starting from Point A and following the sequence A⇨B⇨C⇨D (see following page for illustrations), an ultimate UDL for a 50mm wide beam module is determined to be 1.1 kN/m^2.

The number of beam modules for the total floor UDL of 20.6 kN/m^2 is derived from:

n = 20.6/1.1
= 18.7

Width, b = 18.7 x 50
= 935, say 950mm wide.

Adopt b = 950mm, h = 625mm



Reinforced Concrete Precast Beams Design Chart – (Design Example 3)

Step 3 : Calculate main steel reinforcement

Self-weight of beam = (0.95 x 0.625 + 0.15 x 2x 0.3) x 24
= 16.41 kN/m

At mid-span, M = [1.4 x 16.41 + 20.6 x 9.6] $8^2/8$
= 1765.9 kNm

h = 625mm, d say 515mm

$M / bd^2 f_{cu}$ = 1765.9 x 10^6 / (950 x 515^2 x 35)
= 0.20

Refer to Figure 2.26, for $M/bd^2 f_{cu}$ = 0.20,

ρ_s/f_{cu} = 6.3 x 10^{-4}
ρ_s = 0.022
A_s = 10764 mm^2
Use 10T32 + 6T25 (A_s = 10987 mm^2)

$\rho_{s'}/f_{cu}$ = 1.25 x 10^{-4}
$\rho_{s'}$ = 0.0044
$A_{s'}$ = 2153 mm^2
Use 8T20 (A_s' = 2514 mm^2)

Step 4 : Design for shear links

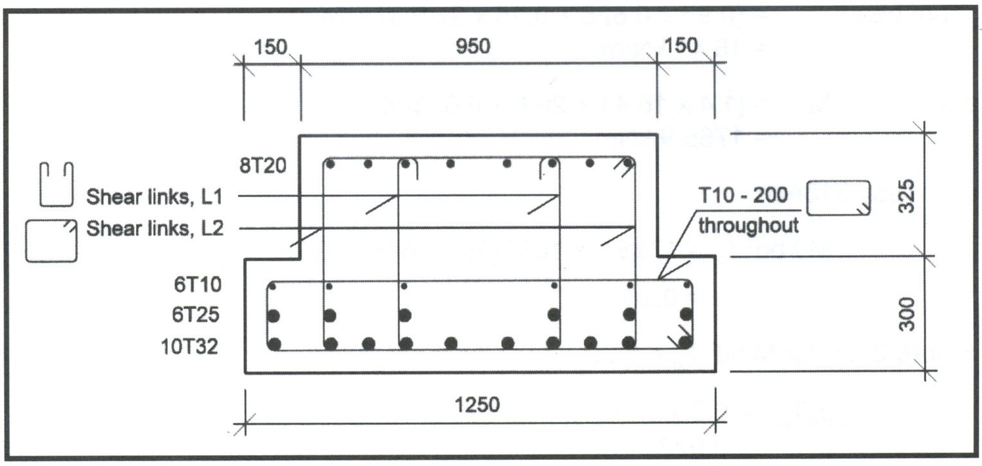
V = 1.4 x 16.41 x 4 + 20.6 x 9.6 x 4
= 882.9 kN

v = 882.9 x 10^3/(950 x 515)
= 1.80 N/mm^2

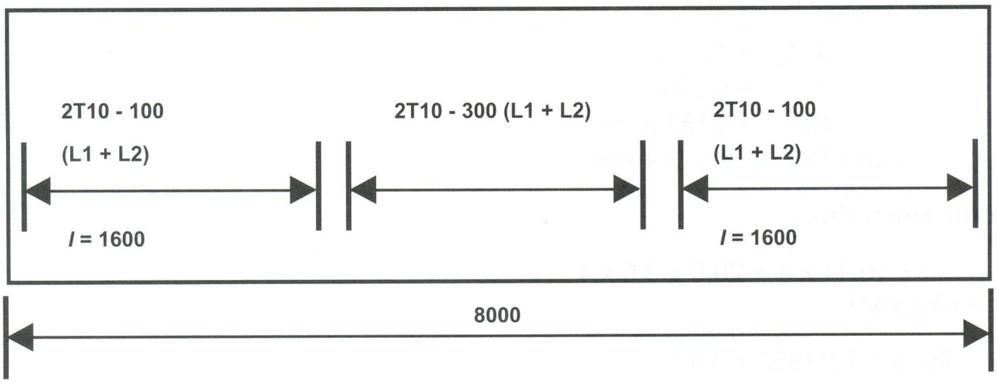
Assume ρ_s at the support to be 30% of mid-span A_s
$\rho_s f_{cu}$ = 0.231
From Figure 2.27, v_c = 0.61N/mm^2

A_{sv}/ s_v = (1.80 - 0.61) x 950/(0.87 x 460)
= 2.82
Use 2T10@100 mm for 1.6 m both ends (A_{sv}/ s_v = 3.14)
Remaining shear links to be 2T10@300 mm

Step 5 : Detailing



Typical Section



Distribution of Shear Links

Design Example 4: Full Section Precast Beam And Unpropped Construction

Re-design the precast beam in Design Example 3 if there is no restriction to the beam depth.

Step 1 : Calculate ultimate floor loading

As per Example 3, ultimate UDL = 20.6 kN/m^2

Step 2 : Determine beam depth

Assume beam width		=	500 mm
No. of beam modules	n	=	500/50
		=	10 nos.
UDL on each beam module		=	20.6/10
		=	2.06kN/m^2

Using Figure 2.22 and following the flow direction in the chart (shown on the following page), the depth of beam is about 825 mm at the intersection at point C. Including 75mm thick topping, the overall beam depth is 900 mm which is a reasonable beam dimension.

Adopt b = 500 mm, h = 825 mm

Step 3 : Calculate main steel reinforcement

Beam s/w = (0.5 x 0.825 + 0.15 x 2 x 0.5) x 24
= 13.5 kN/m

At mid-span, M = $(1.4 \times 13.5 + 20.6 \times 9.6) \times 8^2/8$
= 1733.3 kNm

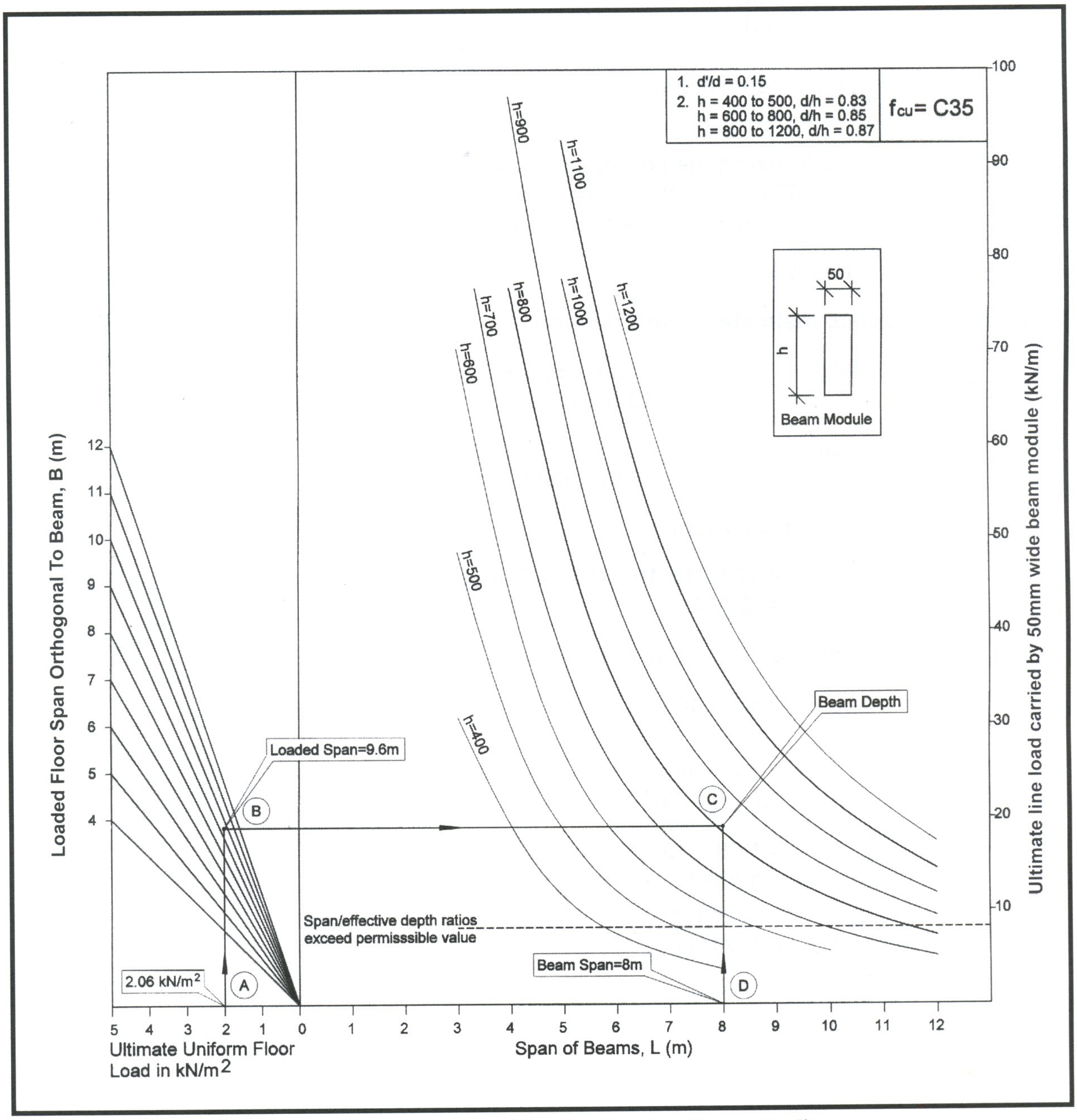
h = 825mm, d say 715mm

$M / bd^2 f_{cu}$ = $1733.3 \times 10^6 / (500 \times 715^2 \times 35)$
= 0.194

Refer to Figure 2.26, for M/bd^2f_{cu} = 0.194,

ρ_s/f_{cu} = 6.0×10^{-4}
ρ_s = 0.021
A_s = 7508 mm^2
Use 8T32 + 4T20 (A_s = 7690 mm^2)

ρ_s'/f_{cu} = 1.06×10^{-4}
ρ_s' = 0.0037
A_s' = 1326 mm^2
Use 5T20 (A_s' = 1571 mm^2)



Reinforced Concrete Precast Beams Design Chart – (Design Example 4)

Step 4 : Design for shear links

$V = (1.4 \times 13.5 + 20.6 \times 9.6) \times 4$
$= 866.6$ kN

$\nu = 866.6 \times 10^3/(500 \times 715)$
$= 2.42$ N/mm^2

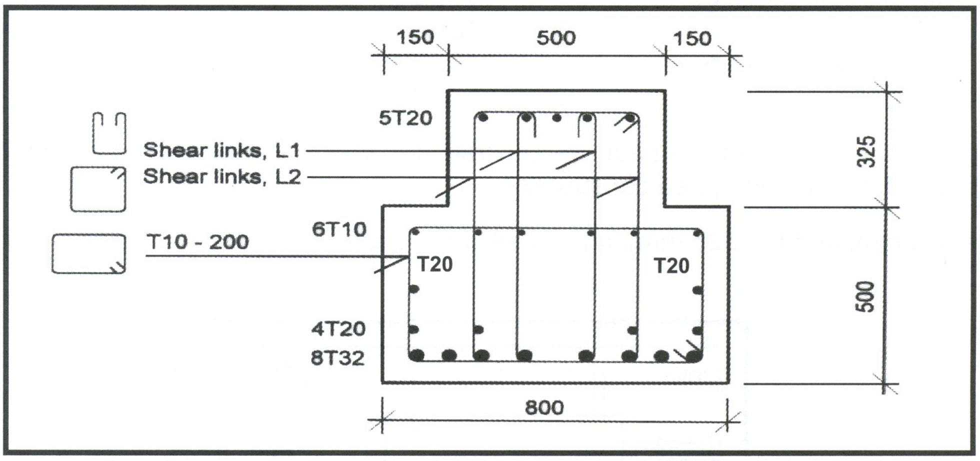
Assume ρ_s, to be 30% of mid-span A_s
$\rho_s f_{cu} = 0.22$

From Figure 2.27, $\nu_c = 0.60$ N/mm^2

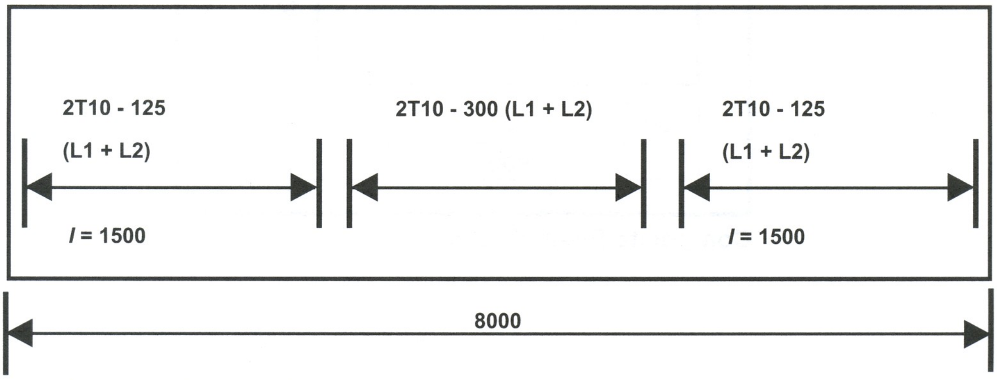
$A_{sv}/s_v = (2.42 - 0.60) \times 500/(0.87 \times 460)$
$= 2.27$

Use 2T10@125 mm for 1.5 m both ends ($A_{sv}/s_v = 2.51$)
Remaining shear links to be 2T10@300 mm

Step 5 : Detailing



Typical Section



Distribution Of Shear Links

Design Example 5: Semi Precast Beams And Unpropped Construction

Design the precast main beams which are to be semi-precast and unpropped during installation. The beams are designed to behave continuous at final stage. The design concrete grade for in-situ topping is C35.

A. At Installation stage

Step 1 : Calculate ultimate floor loading

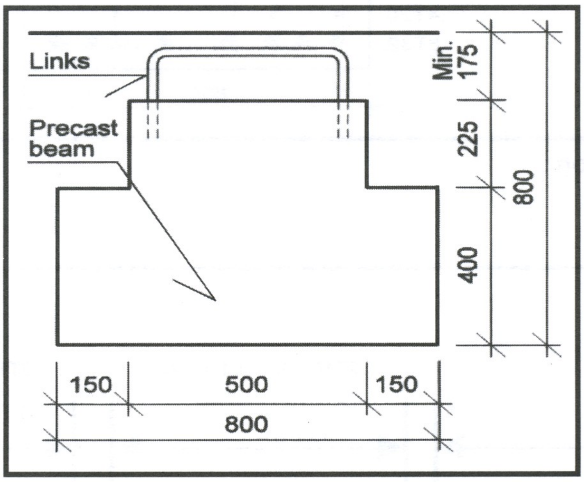
Dead load :		
HC slabs (jointed weight)	=	4.50 kN/m^2
Topping (75 mm thk)	=	1.80 kN/m^2
Total	=	6.30 kN/m^2
Allow live load (construction)	=	1.50 kN/m^2
Ultimate UDL = 1.4DL + 1.6LL	=	11.22 kN/m^2

Step 2 : Determine beam depth and width

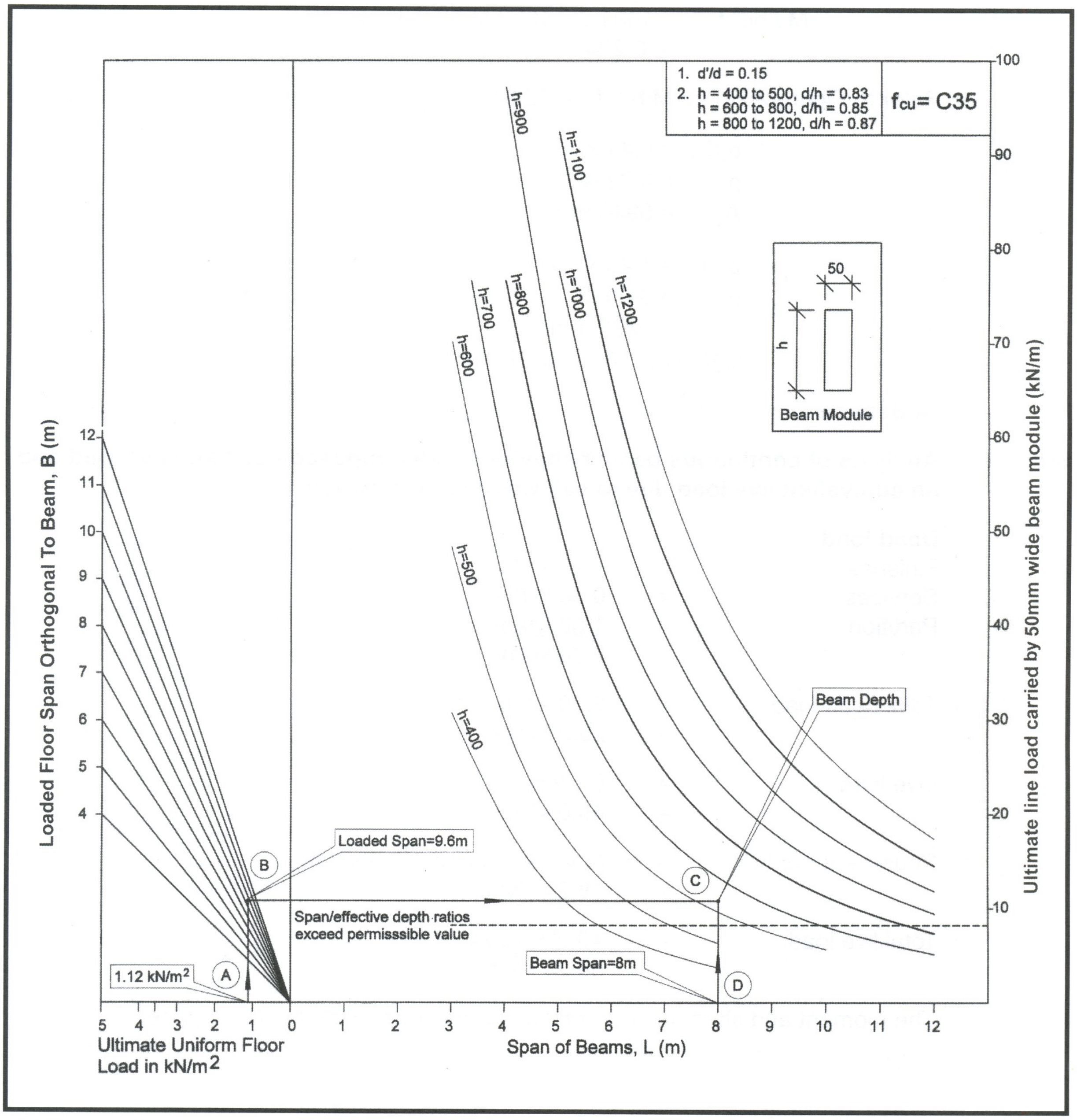
Assume beam width	=	500 mm
No. of beam modules, n	=	500/50
	=	10 nos.
UDL on each module	=	11.22/10
	=	1.12 kN/m^2

Using Figure 2.22 (see following page for illustrations) where the minimum semi-precast beam depth is 625 mm for an unpropped construction.

Adopt b = 500 mm, and h = 625 mm and overall beam depth = 800 mm as shown below.



Composite Beam Section



Reinforced Concrete Precast Beams Design Chart – (Design Example 5)

Step 3 : **Calculate main tension steel requirement at installation**

Beam s/w		=	(0.5 x 0.8 + 0.15 x 2 x 0.4) x 24
		=	12.5 kN/m
At mid-span,	M	=	(1.4 x 12.5 + 11.22 x 9.6) x $8^2/8$
		=	1001.7 kNm

h = 625 mm, d say 530 mm

$M / bd^2 f_{cu}$ = 1001.7 x 10^6 / (500 x 530^2 x 35)
= 0.204

Refer to Figure 2.26, for $M/bd^2 f_{cu}$ = 0.204,

ρ_s/f_{cu} = 6.41 x 10^{-4}
ρ_s = 0.0224
A_s = 5945 mm^2

ρ_s'/f_{cu} = 1.40 x 10^{-4}
ρ_s' = 0.0049
A_s' = 1298 mm^2
Use 5T20 (A_s' = 1571 mm^2)

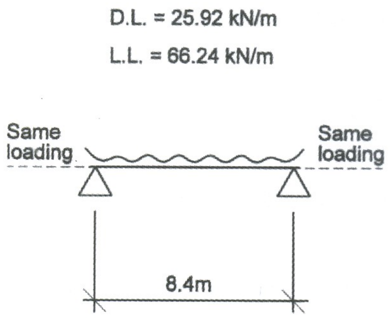
B. **At Service**

Step 4 : **Analysis of continuous beam behaviour under imposed dead and live load and an equivalent live load due to self weight of the structure**

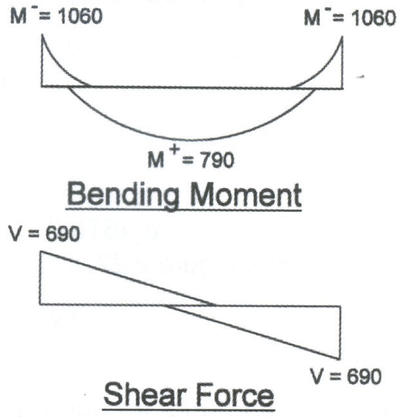
Dead load :

Finishes	=	1.20 kN/m^2
Services	=	0.50 kN/m^2
Partition	=	1.00 kN/m^2
		2.70 kN/m^2
Total dead load	=	2.70 x 9.6
	=	25.92 kN/m
Live load	=	5 x 9.6
	=	48.0 kN/m
Equivalent live load	=	(0.4/1.6) x (12.5 + 6.3 x 9.6)
	=	18.24 kN/m
Total live load	=	48.0 + 18.24
	=	66.24 kN/m

The moment and shear in final condition are shown in the following page.



Loading On Continuous Beam



Distribution Of Bending Moment And Shear (Typical Internal Beam)

Step 5 : Design of main tension steel

Span :

b = 500mm, h = 800 mm, d ≈ 700 mm

M^+ = 790 kNm

$M^+ / bd^2 f_{cu}$ = 790 x 10^6 / 500 x 700^2 x 35
= 0.092

Refer to Figure 2.26, for $M/bd^2 f_{cu}$ = 0.092,

ρ_s/f_{cu} = 2.6 x 10^{-4}
ρ_s = 0.0091
A_{s2} = 3185 mm²

Total A_s = A_{s1} (From Step 3) + A_{s2}
= 5945 + 3185
= 9130 mm²

Use 9T32 + 4T25 (A_s = 9201 mm²)

Note : The above summation of A_s from installation and final stage for the mid-span main steel may appear to contradict Figure 2.20. However, the final result will be the same if a detail crack section analysis is carried out at both the installation and at the final stage and with the steel service stress limited to $5/8f_y$ (= 287 N/mm²). The calculation in Step 5 is

Support :

b = 800mm, h = 800 mm, d ≈ 700 mm

M^- = 1060 kNm

$M^- / bd^2 f_{cu}$ = 1060 x 10^6 / (800 x 700^2 x 35)
= 0.077

Refer to Figure 2.26, for $M/bd^2 f_{cu}$ = 0.077

ρ_s/f_{cu} = 2.1 x 10^{-4}
ρ_s = 0.00735
A_s = 4116 mm²

Use 5T32 (A_s = 4021 mm²)

Step 6 : **Design shear links**

At support, $V = 690 + (12.5 + 6.30 \times 9.6) \times 4.0 \times 1.0$ (refer Figure 2.18)
$= 981.9$ kN

$\nu = 981.9 \times 10^3/(500 \times 700)$
$= 2.81$ N/mm^2

ρ_s (5T32) $= 0.0115$ (b=500, refer to Figure 2.16)
From Figure 2.27, $\nu_c = 0.74$ N/mm^2

$A_{sv}/ s_v = (2.81 - 0.74) \times 500/(0.87 \times 460)$
$= 2.59$

Step 7 : **Shear links for composite action**

Support : contact length, $\mathbf{l}_e \approx 0.2 \times 8.4 - 0.2$
$= 1.48$ m

$A_{sv}/ s_v = A_s/\mathbf{l}_e$
$= 4116/1480$
$= 2.78$

Span : contact length, $\mathbf{l}_e \approx 0.35 \times 8.4$
$= 2.94$ m

Contact width, $b_e = 500$ mm

Neutral axis depth, $\chi = 0.87 f_y A_s/(0.45 f_{cu} \times b_e \times 0.9)$
$= (0.87 \times 460 \times 4116) / (0.45 \times 35 \times 800 \times 0.9)$
$= 145.2$ mm < 175 mm

Hence horizontal shear force,
$V_h = 0.45 f_{cu}\, b_e\, \chi$
$= 0.45 \times 35 \times 500 \times 145.2 \times 10^{-3}$
$= 1143.5$ kN

Average horizontal shear stress in mid-span,
$\nu_h = V_h/b_e\mathbf{l}_e$
$= 1143.5 \times 10^3/(500 \times 2940)$
$= 0.78$ N/mm^2 < 1.9 N/mm^2 (Table 5.5 Part 1, BS 8110)

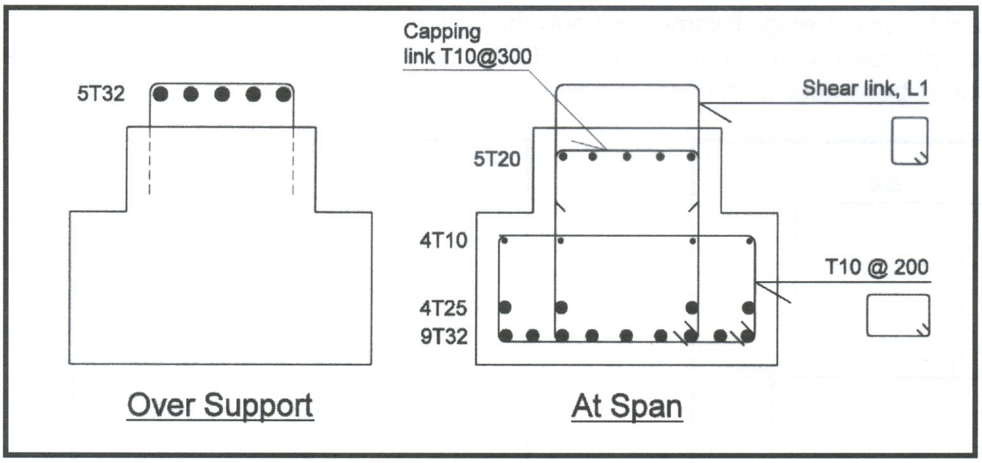
Step 8 : **Shear links provision**

Support : $A_{sv}/s_v = 2.78$ (the greater A_{sv}/s_v of Step 5 and Step 6)

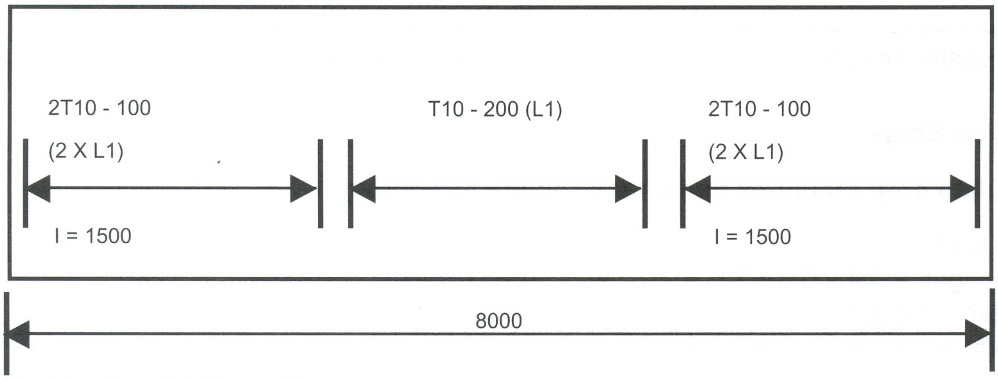
Use 2T10-100 links for 1.5 m both ends

Span : use nominal links T10-200

Step 9 :   Detailing



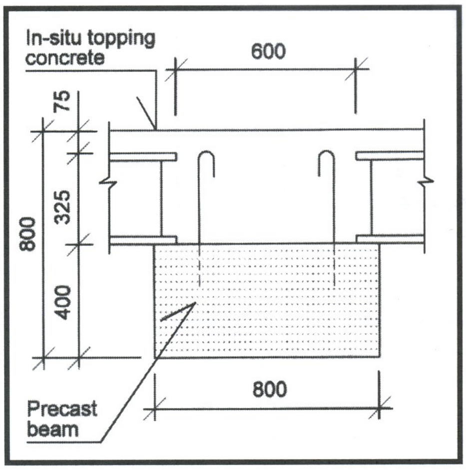
Typical Beam Section



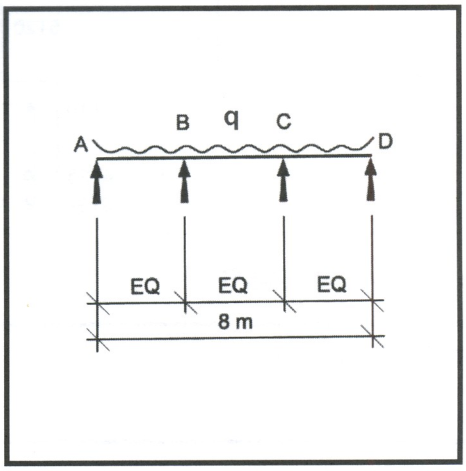
Distribution Of Shear Links

Design Example 6 : Semi Precast Beam And Propped Construction

Re-design the precast beam in Design Example 5 if only the bottom 400 mm deep section is precast. The beams are to be propped at 1/3 point during the floor slab installation. The beams are to be designed for continuous behaviour at the final condition. Topping concrete used is C35 concrete.



Typical Composite Section

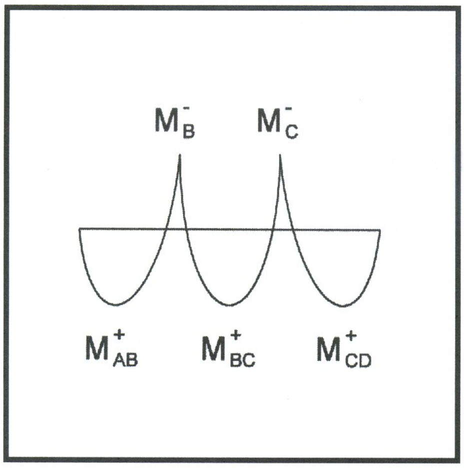


Propping Of Precast Beam

A. At Installation Stage

Step 1 : Calculate prop reactions and check section strength

Dead load :		
Self weight = 0.4 x 0.8 x 24	=	7.68 kN/m
HC slab = 4.5 x 9.0	=	40.50 kN/m
Topping = 1.8 x 9.6 + 0.6 x 0.4 x 24	=	23.04 kN/m
Total	=	71.22 kN/m
Live load (construction) = 1.5 x 9.6	=	14.40 kN/m
Ultimate load = 1.4 x DL + 1.6 x LL	=	122.75 kN/m
Propped reaction at A and D		
Dead load = 0.4 x 8/3 x 71.22	=	76.0 kN
Live load = 0.4 x 8/3 x 14.40	=	15.4 kN
Propped reaction at B and C		
Dead load = 1.1 x 8/3 x 71.22	=	208.9 kN
Live load = 1.1 x 8/3 x 14.40	=	42.2 kN



Bending Moment In Propped Precast Beam

Step 2: Design for installation

Support :

$M_B^- = M_C^- = 0.1 \times (8/3)^2 \times 122.75 = 87.3$ kNm

$M^- / bd^2 f_{cu} = 87.3 \times 10^6 / (800 \times 290^2 \times 35) = 0.037$

From Figure 2.26, ρ_s/f_{cu} (min.) $= 0.98 \times 10^{-4}$

$\rho_s = 0.00343$

$A_{s1} = 796$ mm²

Use 4T16 ($A_s = 804$ mm²)

Span :

$M_{AB}^+ = M_{CD}^+ = 0.08 \times (8/3)^2 \times 122.75 = 69.8$ kNm

$\rho_s = 0.0027$

$A_{s1} = 626$ mm²

M_{BC}^+ is not critical and assumes $A_{s1} = 796$ mm²

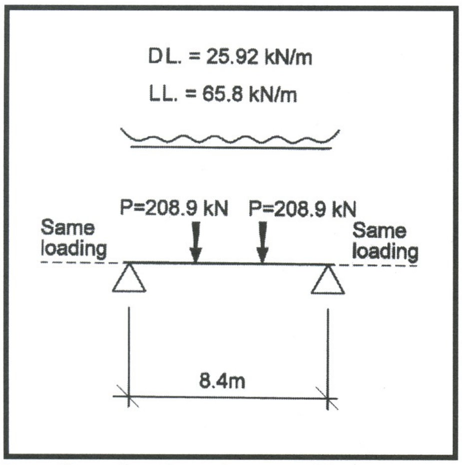
B. At Service

Step 3 : Analysis of beam under imposed dead and live load and reverse prop reactions

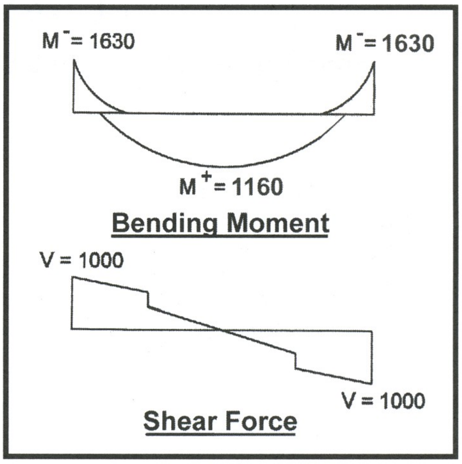
Dead load:

Finishes = 1.2 x 9.6	=	11.52 kN/m
Services = 0.5 x 9.6	=	4.80 kN/m
Partition = 1.0 x 9.6	=	<u>9.60 kN/m</u>
Total DL	=	25.92 kN/m

Live load = 5 x 9.6	=	48.0 kN/m
Equivalent L.L. = (0.4/1.6) x 71.22	=	<u>17.8 kN/m</u>
Total LL	=	65.8 kN/m
Propping reaction P = dead load (only due to s/w of structure)	=	208.9 kN



Loading On Continuous Beam



Distribution Of Bending Moment And Shear (Typical Internal Beam)

Step 4 : **Design of main tension steel**

Span : b = 600mm, h = 800mm, d = 700mm

$M^+ = 1160$ kNm

$M^+ / bd^2 f_{cu} = 1160 \times 10^6 / (600 \times 700^2 \times 35)$
$= 0.113$

From Figure 2.26, $\rho_s/f_{cu} = 3.4 \times 10^{-4}$

$\rho_s = 0.0119$

$A_{s2} = 4998\ mm^2$

Total $A_s = A_{s1}$ (from step 2) + A_{s2}
$= 626 + 4998$
$= 5624\ mm^2$

Use 7T32 ($A_s = 5629\ mm^2$)

Support : b = 800mm, h = 800mm, d = 700mm

$M^- = 1630$ kNm

$M^- / bd^2 f_{cu} = 1630 \times 10^6 / (800 \times 700^2 \times 35)$
$= 0.119$

From Figure 2.26, $\rho_s/f_{cu} = 3.49 \times 10^{-4}$

$\rho_s = 0.0122$

$A_{s2} = 6832\ mm^2$

Use 6T32 + 4T25 (A_s= 6789 mm^2, marginally under provided, OK)

Step 5 : **Design for shear links**

At support, V = 1.0 x (Step 1) + (Step 3)
= 1.0 X 76.0 + 1000
= 1076.0 kN

$\nu = 1076.0 \times 10^3/(600 \times 700)$ (b=600, refer to Figure 2.19)
$= 2.56\ N/mm^2$

ρ_s (6T32 + 4T25) = 6789/(600 x 700)
= 0.0162

$\rho_s f_{cu} = 0.567$

From Figure 2.27, $\nu_c = 0.83\ N/mm^2$

$A_{sv}/ s_v = (2.56 - 0.83) \times 600/(0.87 \times 460)$
$= 2.59$

Step 6 : **Design for composite action**

Support : Contact width, b_e = 600 mm

Contact length, l_e = 0.2 x 8.4 - 0.2
= 1.48 m

A_{sv}/s_v = A_s/l_e
= 6832/1480
= 4.62

Span : Contact length, l_e ≈ 0.35 x 8.4
= 2.94 m

A_{sv}/s_v = A_s/l_e
= 5624/2940
= 1.91

Step 7 : **Provide shear links for both composite action and shear resistance, whichever is greater**

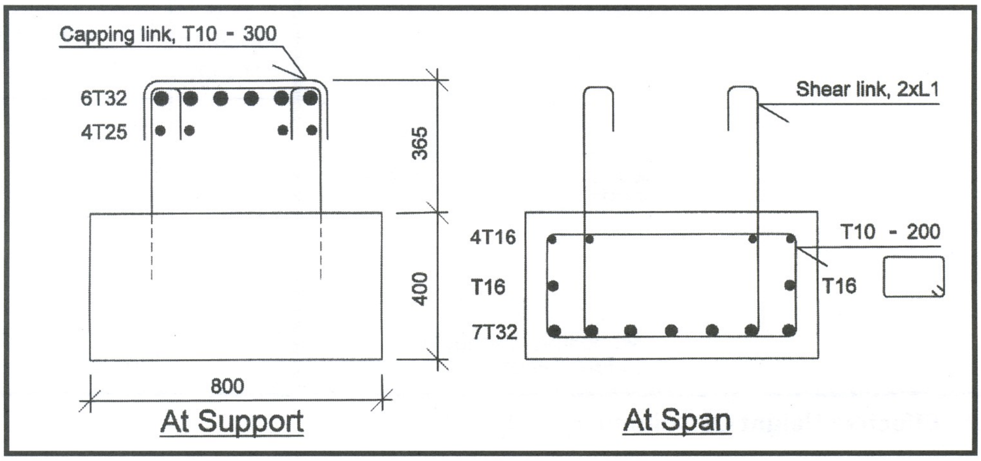
Support : A_{sv}/s_v = 4.62

Use 2T13 - 100 c/c links for 1.5 m at end span (A_{sv}/s_v = 5.31)

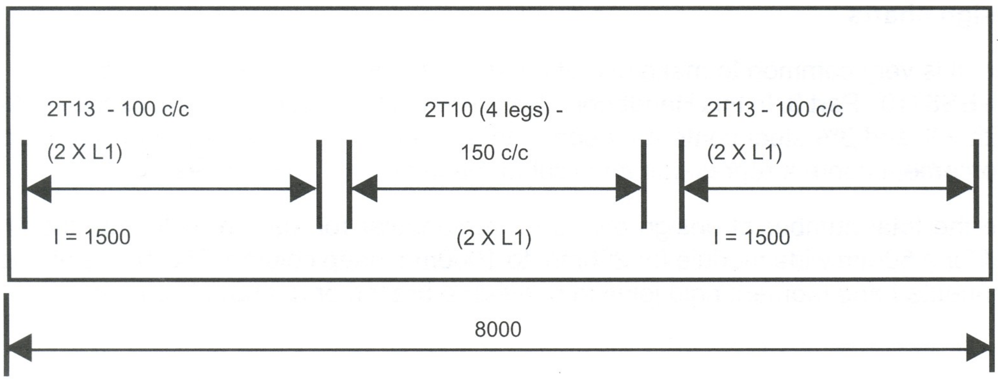
Span : A_{sv}/s_v = 1.91

Use 2T10 - 150 c/c links for mid-span (A_{sv}/s_v = 2.09)

Step 8 : **Detailing**



Typical Beam Section



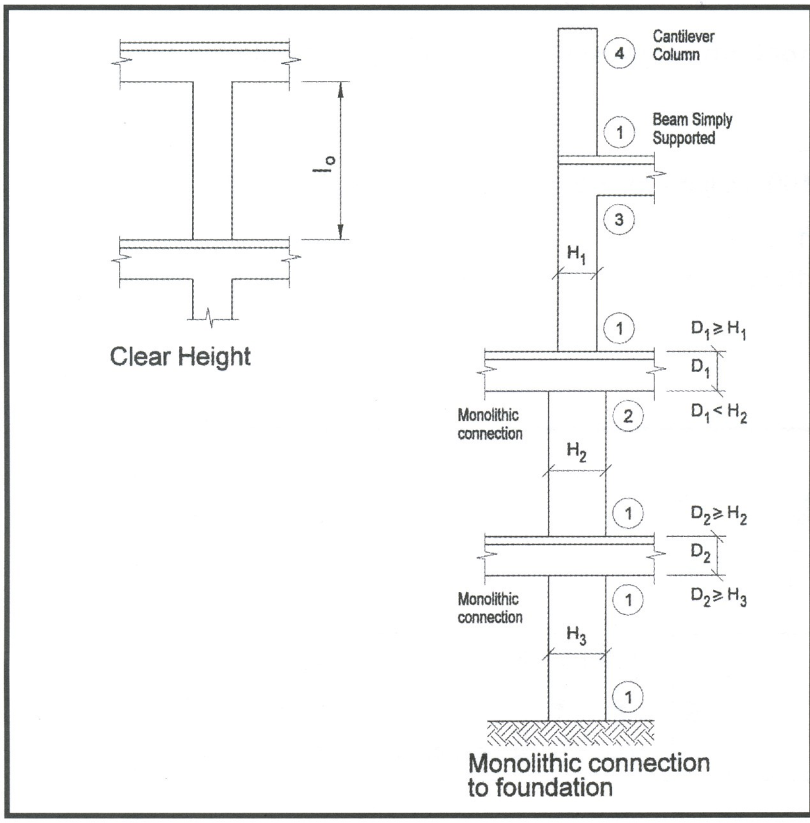
Distribution Of Shear Links

2.4 Design Of Precast Concrete Columns

The design of precast concrete columns is similar in approach to those for in-situ columns. The design methods complying to the code requirements are well documented in most standard texts and will not, therefore, be elaborated further in this section.

In the design of precast concrete columns, the designer should be conversant with the various connection methods used in jointing column-to-foundation, column-to-column and column-to-beam in order to achieve the desired joint behaviour which could be either moment-rigid or pin-connected.

Particular attention should be given to ensure that the connection details will not jeopardise the structural stability of the building. In addition, the columns must have sufficient capacity to withstand failure from buckling due to slenderness effect. A summary of ß values for braced and unbraced columns in accordance with Part 1 clause.3.8.1.6, of the Code is shown in Figure 2.29.



Braced Columns

End condition at top	End condition at bottom 1	2	3
1	0.75	0.80	0.90
2	0.80	0.85	0.95
3	0.90	0.95	1.00

Unbraced Columns

End condition at top	End condition at bottom 1	2	3
1	1.2	1.3	1.6
2	1.3	1.5	1.8
3	1.6	1.8	-
4	2.2	-	-

Value of β

Figure 2.29 Effective Height Of Column, $l_e = \beta\, l_o$

2.4.1 Design charts

In practice, it is very common to make use of charts in the design of columns; a collection of which is found in BS8110 : Part 3. In this Handbook, design charts for rectangular and circular columns are shown with 2% and 3% steel content for concrete grades f_{cu} = 35, 45 and 50 N/mm^2 respectively. The reinforcement content represents a typical range in precast column design.

To reduce the total number of design charts for rectangular (or square) columns, the charts are presented for a 50mm wide module for 200mm to 1000mm deep column. The charts are applicable for pin-connected and moment rigid jointing of either a braced or unbraced column.

The load eccentricities shown in the charts arise from:

1. actual design eccentricity such as beam supported by corbel,
2. h/20 or minimum 20mm,
3. additional eccentricity due to column slenderness effect as determined in Part 1 clause. 3.8.3.,
4. eccentricity due to framing moment in a moment rigid column-beam connection where the eccentricity is calculated as e = M/N where N is the total column load at the level being considered.

For columns under biaxial bending, the enhanced bending moment in either the minor or major axis should be determined in accordance with Part 1, clause 3.8.4.5 of the Code.

In using the design charts, the following steps may be taken:

Step 1. Determine the total ultimate column load, N, at the level being considered.

Step 2. Divide N by n which is a multiple of 50mm module for an assumed or given column width.

Step 3. Determine the load eccentricities as described earlier.

It should be noted that it may not always be possible to obtain the framing moment in a column unless actual column stiffness is used in the analysis. This, however, cannot be done before a reasonable column size is fixed. To overcome this problem, the column is usually sized by assuming a value of bending stresses M/bh^2 in the column which is generally taken to be:

Internal columns	= 1.5 to 2.5N/mm^2
Edge and corner columns	= 3.0 to 4.0N/mm^2

The M/bh^2 graphs are shown in all the column design charts from Figures 2.30 to 2.35 for rectangular/square columns and Figures 2.36 to 2.41 for circular columns.

The design charts for circular columns follow similar approach as that for rectangular columns except the axial load capacities are given for the actual column size. Hence, Step 2 in the design process is omitted.

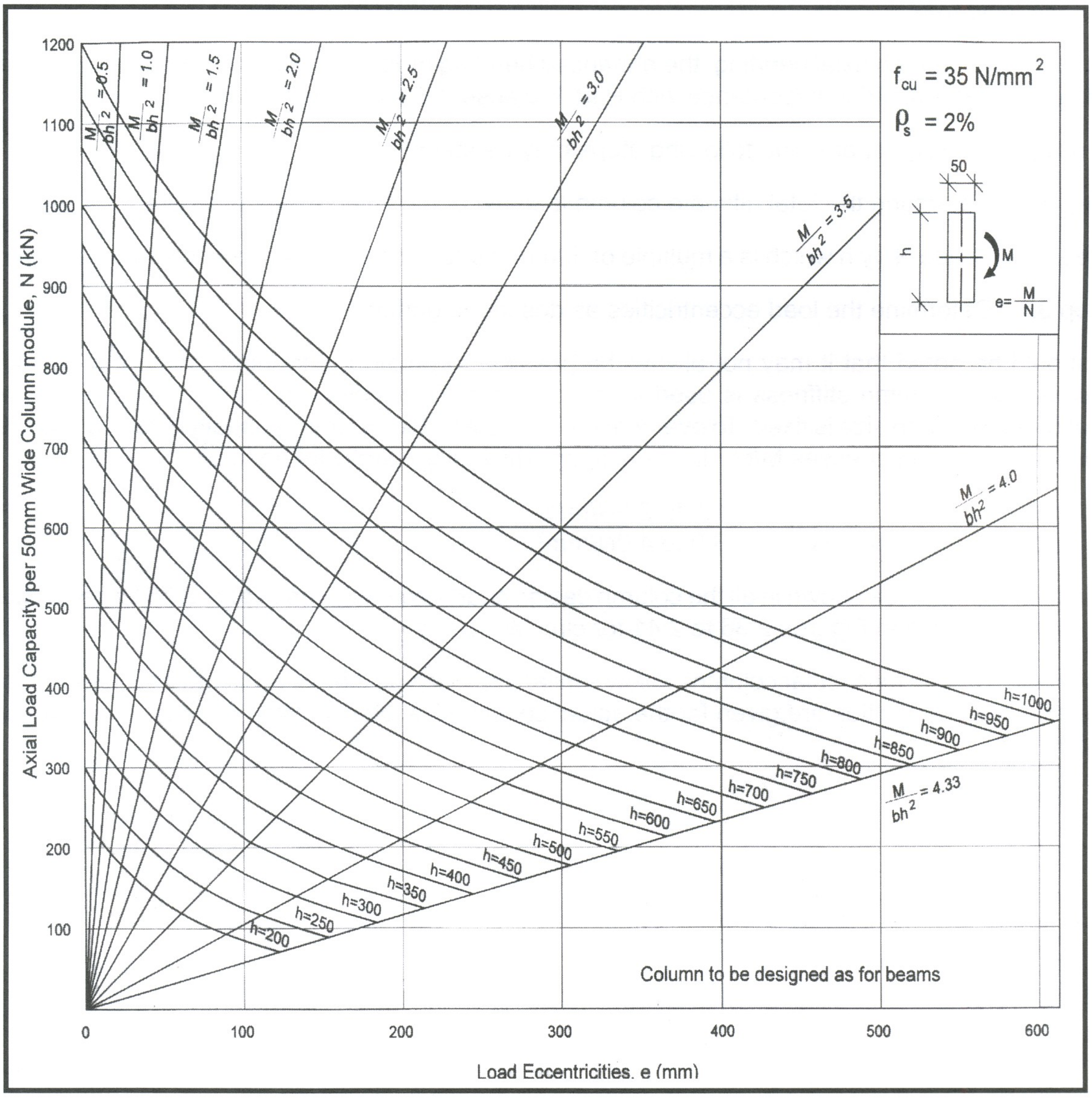


Figure 2.30 Reinforced Concrete Precast Rectangular/Square Column Design Chart For f_{cu}=35N/mm^2, And ρ_s=2%

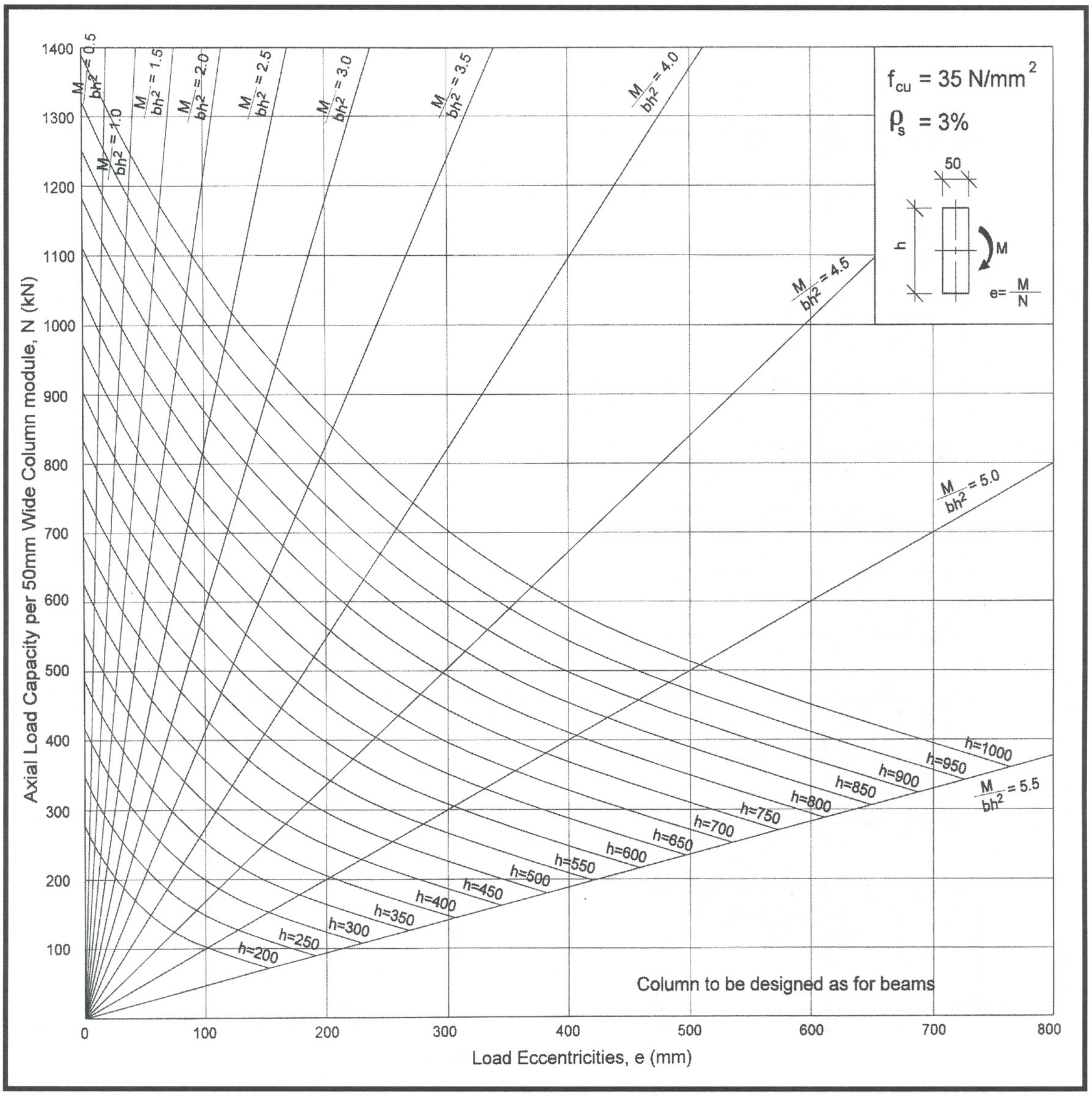


Figure 2.31 Reinforced Concrete Precast Rectangular/Square Column Design Chart For f_{cu}=35N/mm² And ρ_s=3%

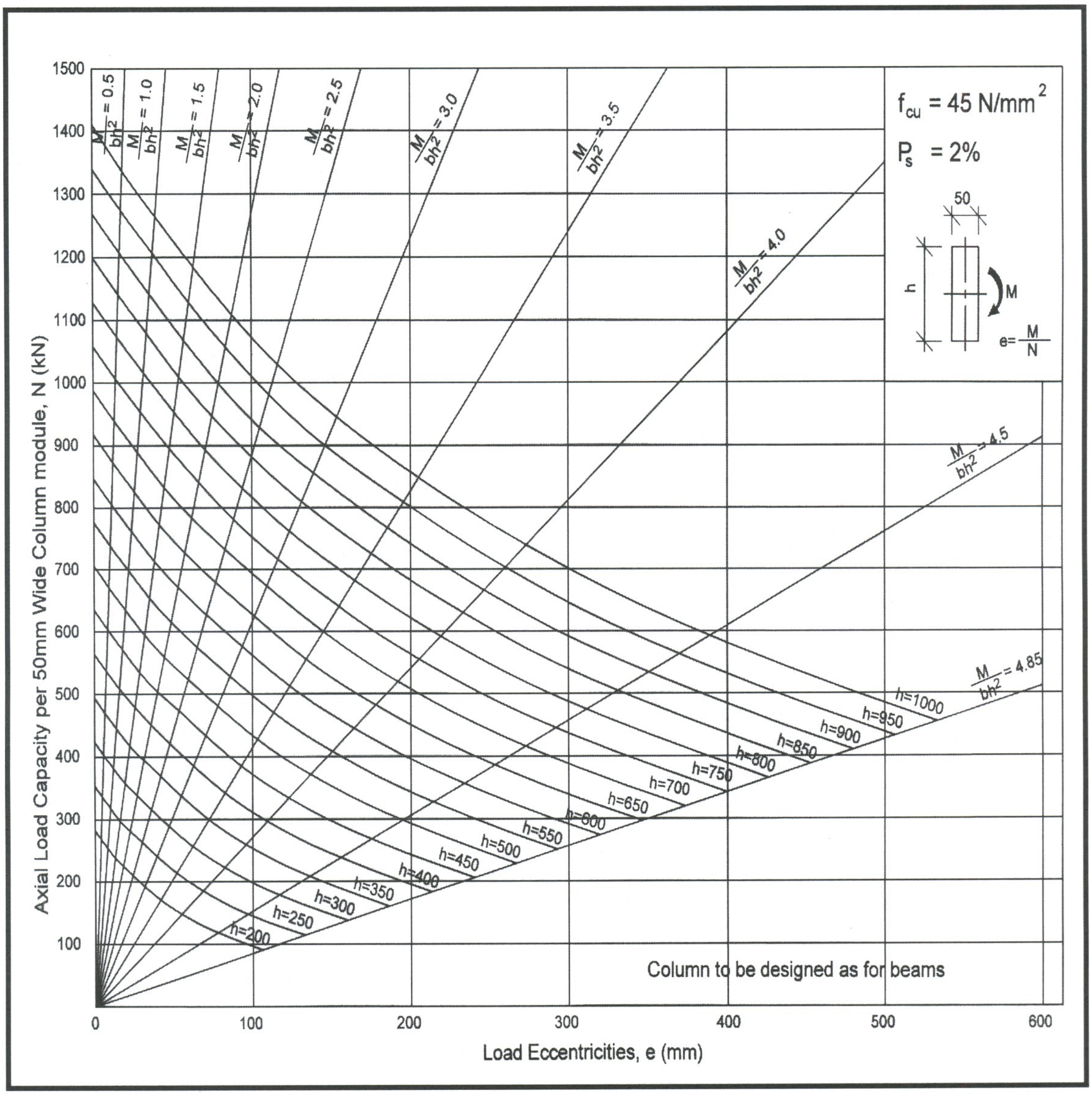


Figure 2.32 Reinforced Concrete Precast Rectangular/Square Column Design Chart For f_{cu}=45N/mm^2 And ρ_s=2%

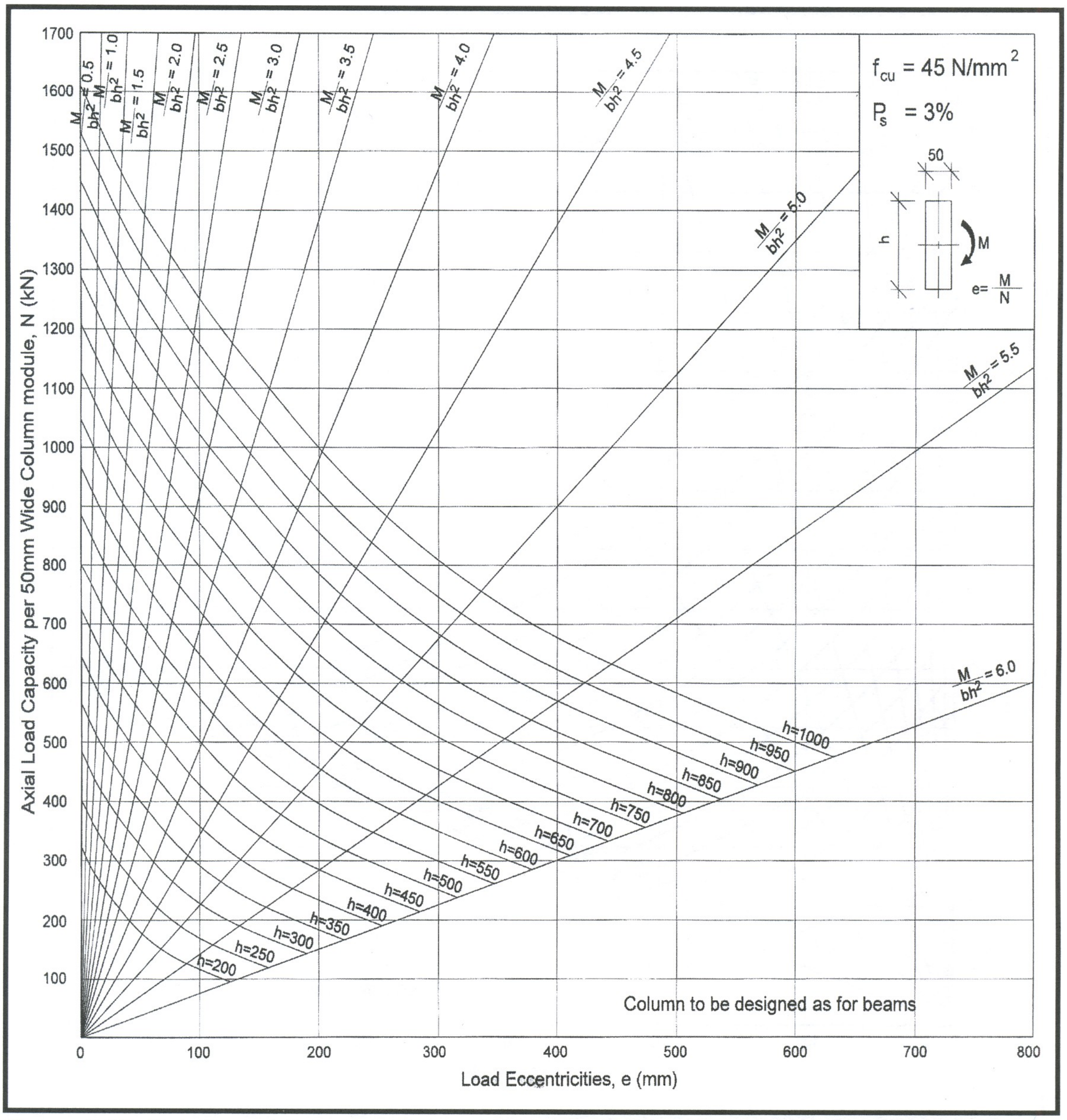


Figure 2.33 Reinforced Concrete Precast Rectangular/Square Column Design Chart For f_{cu}=45N/mm^2 And ρ_s=3%

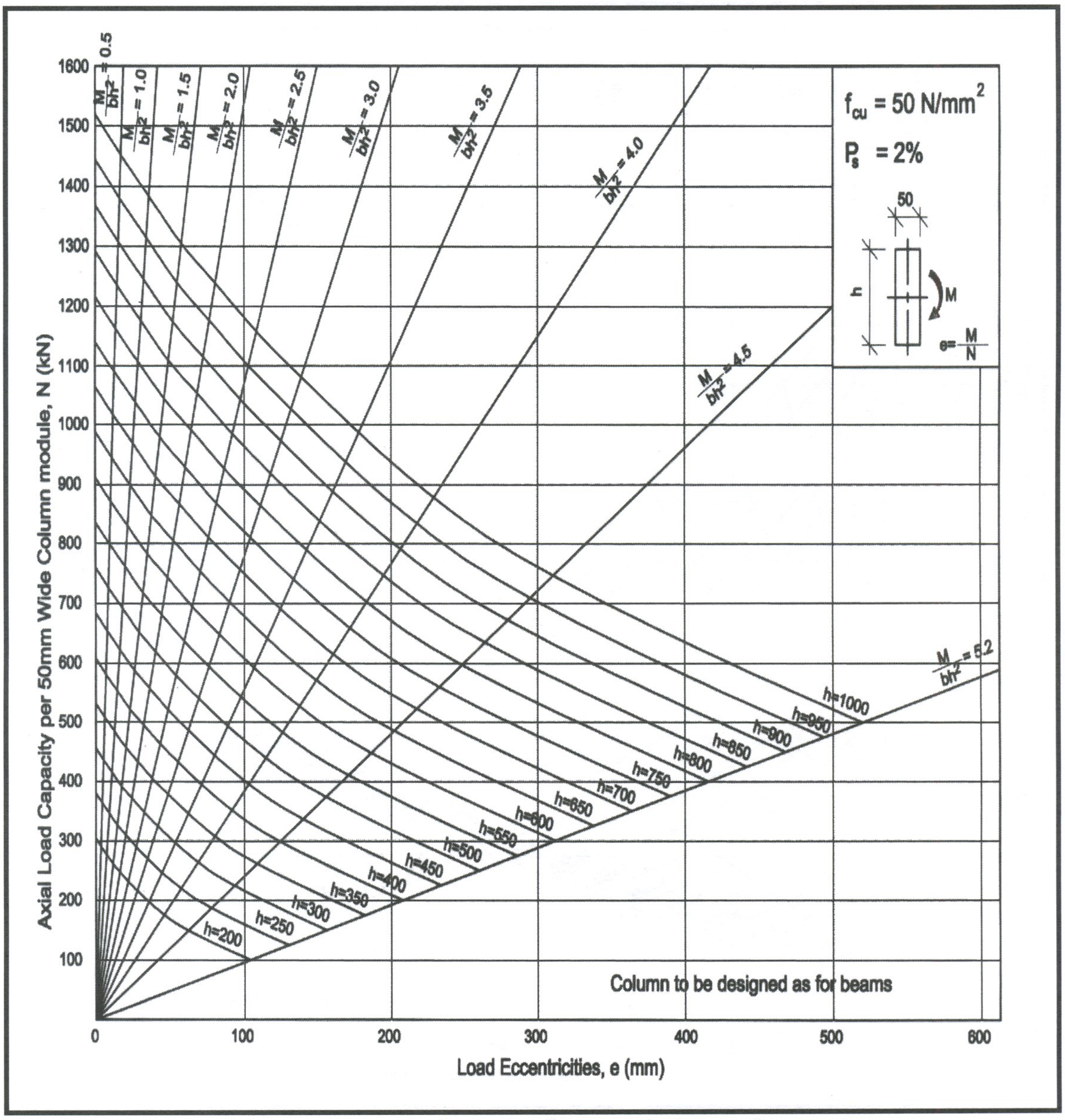


Figure 2.34 Reinforced Concrete Precast Rectangular/Square Column Design Chart For f_{cu}=50N/mm² And ρ_s=2%

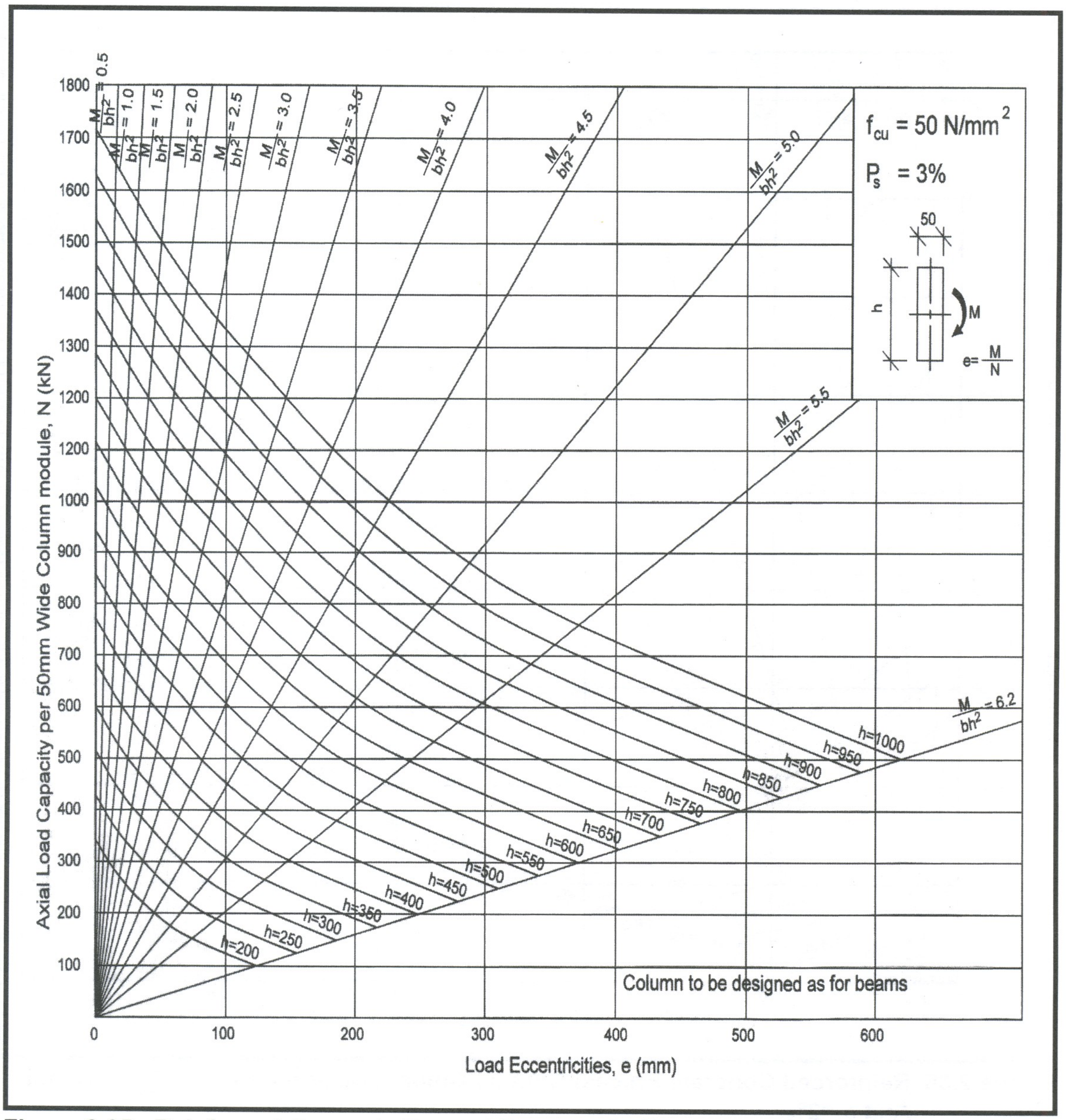


Figure 2.35 Reinforced Concrete Precast Rectangular/Square Column Design Chart For f_{cu}=50N/mm² And ρ_s=3%

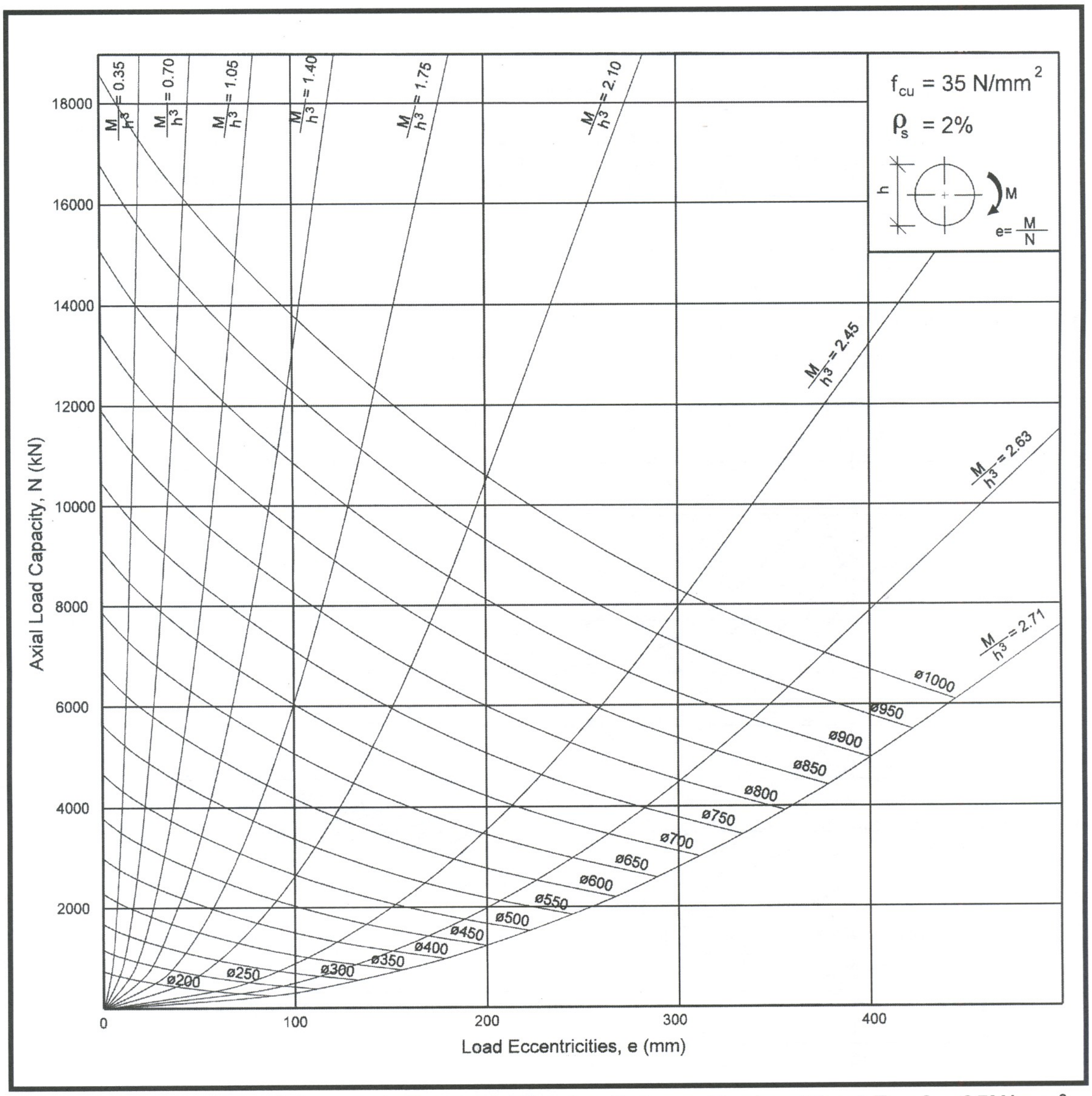


Figure 2.36 Reinforced Concrete Precast Circular Column Design Chart For f_{cu}=35N/mm² And ρ_s=2%

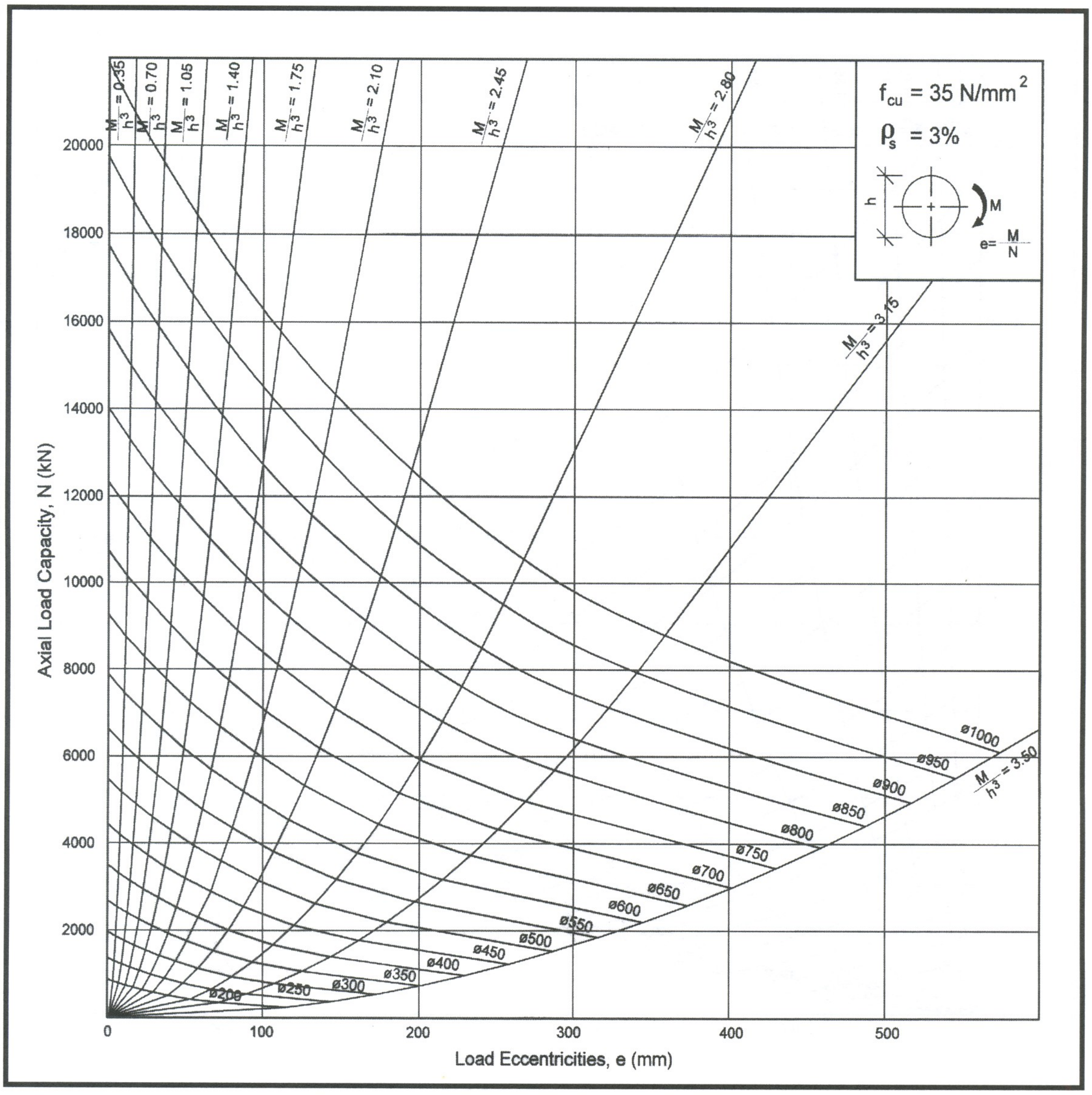


Figure 2.37 Reinforced Concrete Precast Circular Column Design Chart For f_{cu}=35N/mm^2 And ρ_s=3%

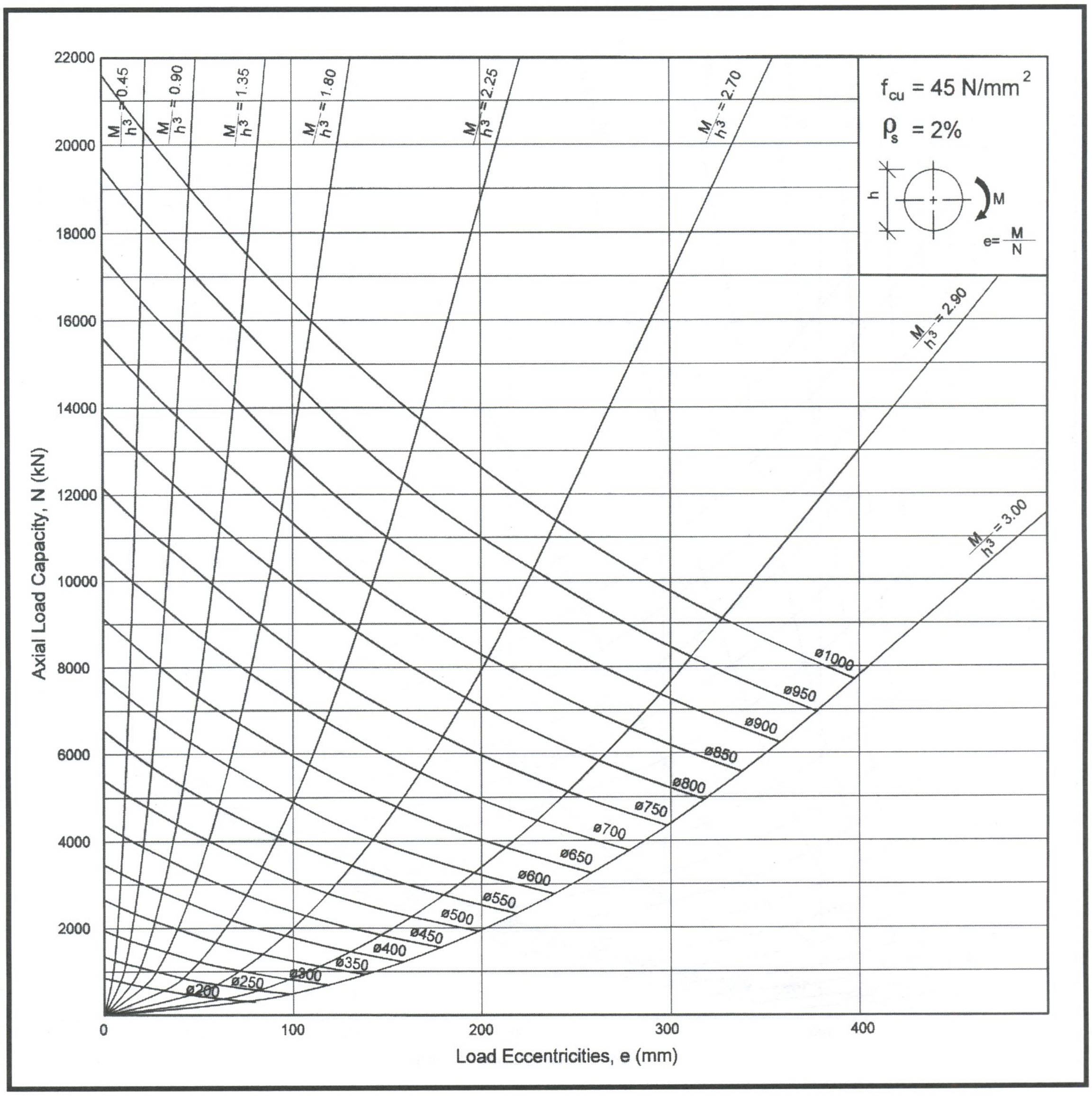


Figure 2.38 Reinforced Concrete Precast Circular Column Design Chart For f_{cu}=45N/mm^2 And ρ_s=2%

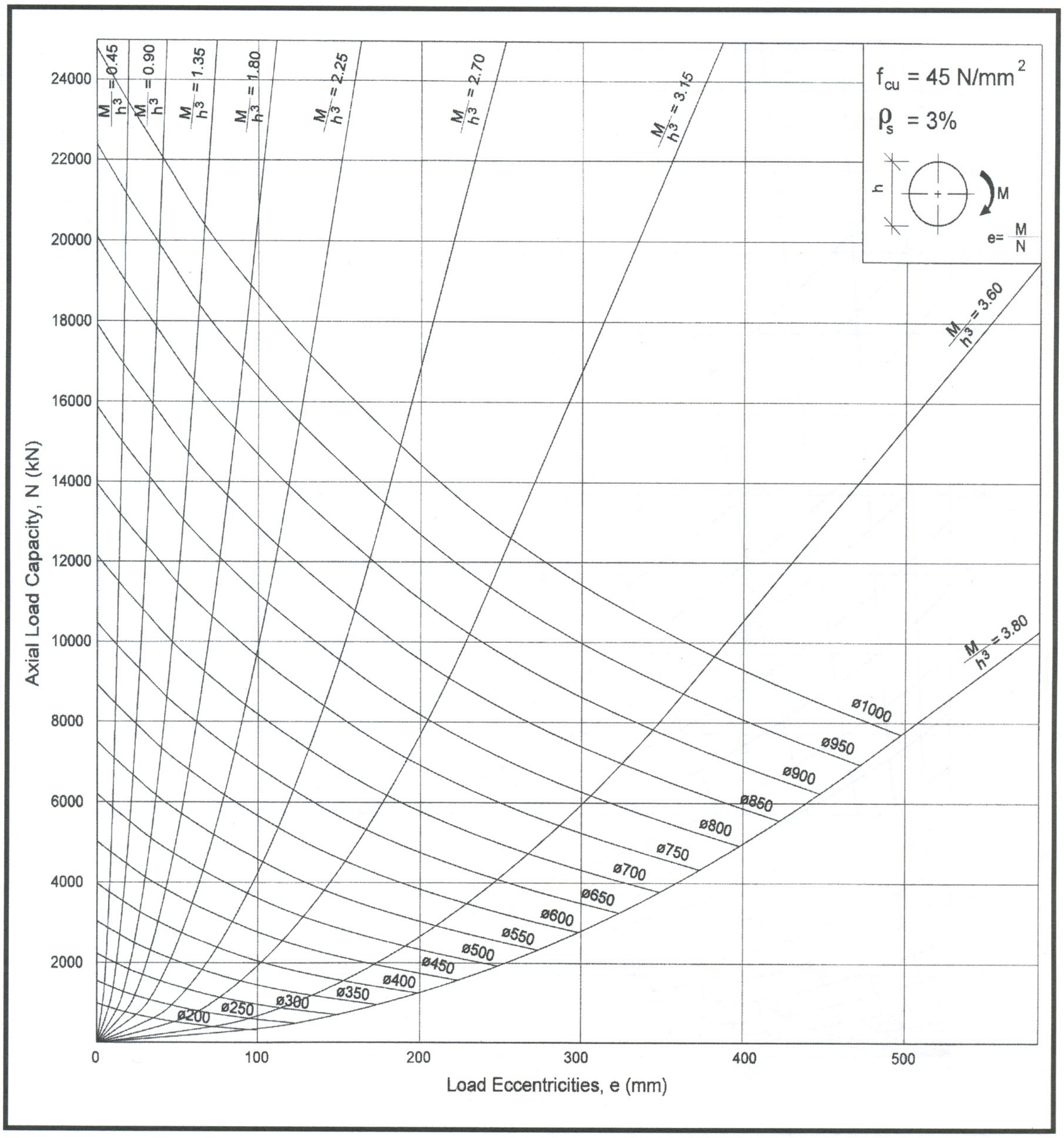


Figure 2.39 Reinforced Concrete Precast Circular Column Design Chart For f_{cu}=45N/mm^2 And ρ_s=3%

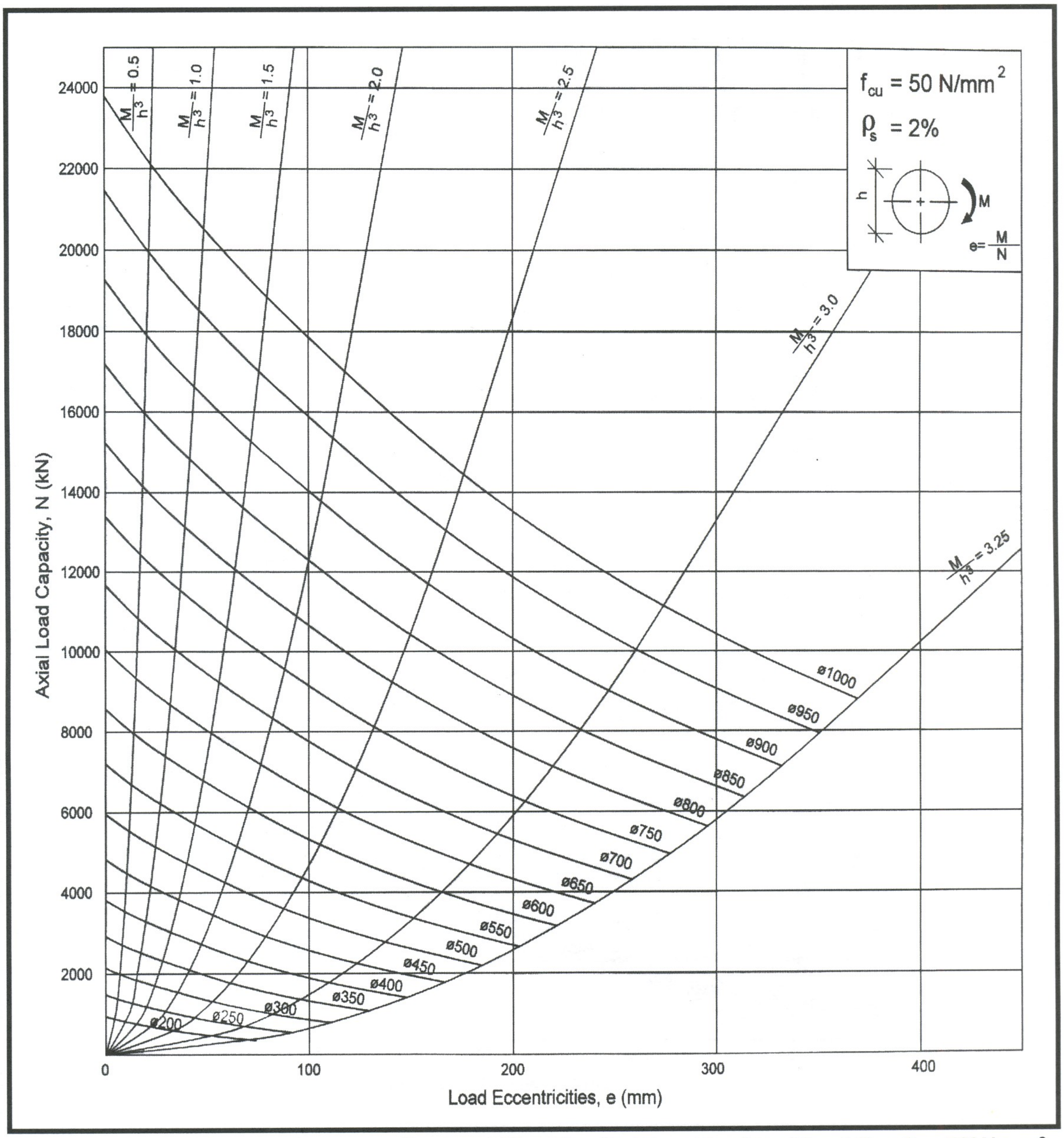


Figure 2.40 Reinforced Concrete Precast Circular Column Design Chart For f_{cu}=50N/mm² And ρ_s=2%

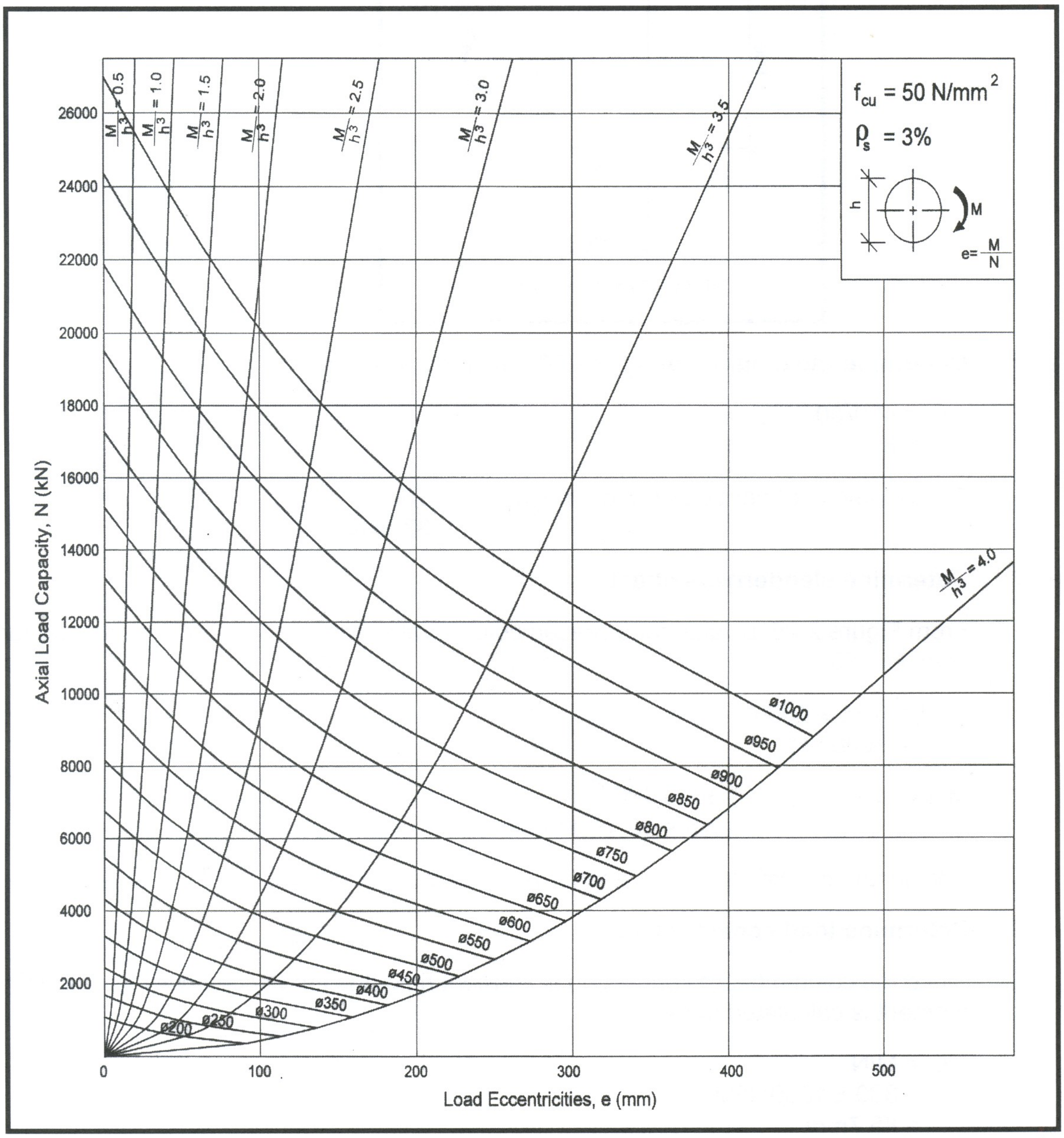
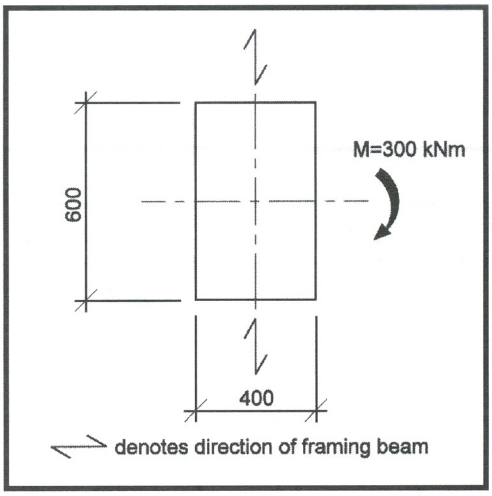


Figure 2.41 Reinforced Concrete Precast Circular Column Design Chart For f_{cu}=50N/mm² And ρ_s=3%

2.4.2 Design examples

Design Example 7: Precast Concrete Column

Determine whether a 400 x 600 precast column is adequate to carry a total ultimate load of 3500 kN and an ultimate framing moment of 300 kNm about the major axis. The column is braced with floor to floor height 4m and design concrete strength f_{cu}=35N/mm^2.



Step 1. Determine the column load for a 50 mm wide module

$n = 400/50$
$= 8$

The column load per 50 mm wide module = 3500/8
= 437.5kN

Step 2. Determine slenderness effect

From Figure 2.29, β value for a braced column with condition 1 at both top and bottom ends is 0.75.

$\therefore l_e = 0.75 \times 4000$
$= 3000$mm

Minor axis $l_e/h = 3000/400$
$= 7.5 < 15$, short column

Additional eccentricity due to slenderness effect is not critical.

Step 3. Determine load eccentricities

Since the column/beam connection is moment rigid, the load eccentricity from framing moment is calculated to be

$e = M/N$
$= 300 \times 1000/3500$
$= 85.7$mm

Step 4. Check adequacy of column size

From Figure 2.30 (f_{cu} = 35N/mm^2, ρ_s = 2%), the required column length for column load of 437.5kN and e = 85.7mm is about h $\approx$ 500mm < 600mm provided.

Therefore the column size 400 x 600mm with 2% reinforcement content is adequate.

2.5 Design Of Precast Concrete Walls

2.5.1 General

The functions of precast concrete walls can be identified by the type of buildings in which they are used :

1. Skeletal frame structures : precast concrete walls are used as non-load bearing in-fill walls and may be designed to provide stability to the building

2. Shear wall structures : the precast walls are reinforced, cantilevered walls designed to carry the vertical loads and horizontal, lateral and in-plane forces.They are used as stabilising elements for the structure and may come in the form of single elements or forming boxes for staircases or lift shafts

In a mixed skeletal frame and shear wall building, both types of walls are provided.

Precast concrete walls may also be used as non-load bearing partitions to replace brickworks so as to achieve better surface quality and minimise site plastering.

The thickness of precast walls varies from 125 mm to 300 mm and is governed either by cranage constraints at the site or factory or by the ultimate shear and load carrying capacity in service. Walls are preferably designed as single elements. For wider walls, it may be necessary to assemble them in separate units with in-situ jointing. Openings for doors, windows and services may be accommodated provided their positions do not interrupt the structural integrity and continuity of the walls. This is particularly important when the wall design is based on the compressive diagonal strut model. Alternative load paths for the vertical and horizontal forces must be considered if the openings are large.

The construction of box walls for staircases and lift shafts can be obtained from individual wall sections or from a complete or partial box in single or part storey high.

There is a wide range of solutions in jointing the walls. These include:

1. in-situ concrete and steel tie
2. welded connections made by fully anchored plates
3. bolting
4. shear keys with or without interlacing steel and
5. simple mortar bedding

The design forces at the vertical and horizontal wall joints primarily consist of compression, tension and shear forces. Design considerations are shown in Chapter 3, section 3.17 in this Handbook.

2.5.2 Design classifications of concrete walls

For design purposes, CP 65 classifies the walls, which are defined as having their length exceeding four times their thickness, into :

1. reinforced concrete walls containing a minimum quantity of steel as given in Part 1, clauses 3.9.3 and 3.12.5.3. The steel is taken into account when determining the strength of the walls, and

2. plain concrete walls in Part 1, clause 3.9.4 where only minimum shrinkage steel is provided. The strength of the walls is based solely on the compression capacity of the concrete.

In addition, the walls are classified as :

1. braced if the walls are supported laterally by floors or other cross-walls

2. unbraced if the walls provide their own stability, such as cantilever walls.

The walls are considered stocky if the slenderness ratio, i.e. l_e/t, does not exceed 15 for a braced wall and 10 for an unbraced wall. Otherwise, the walls are considered as slender.

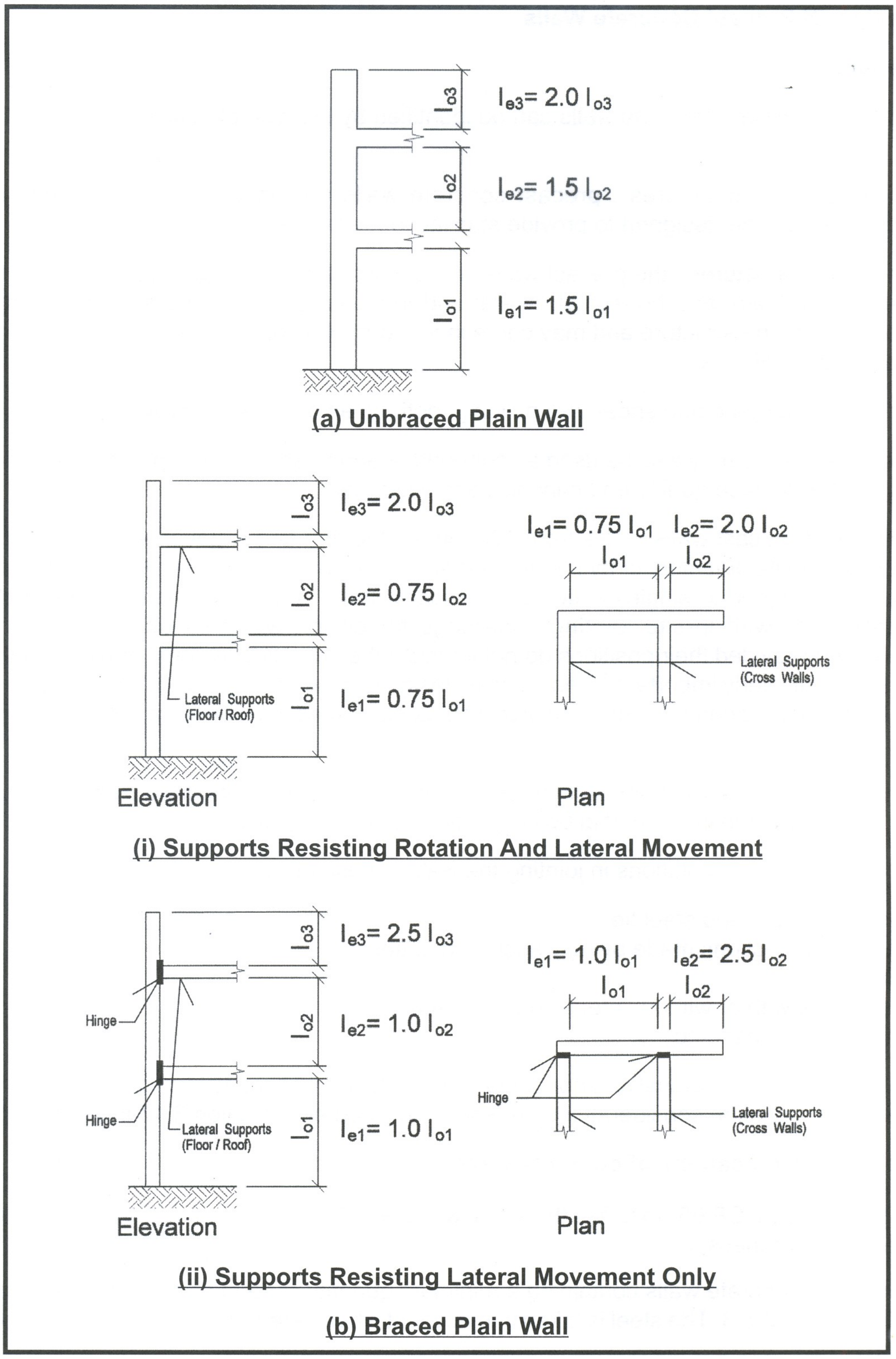


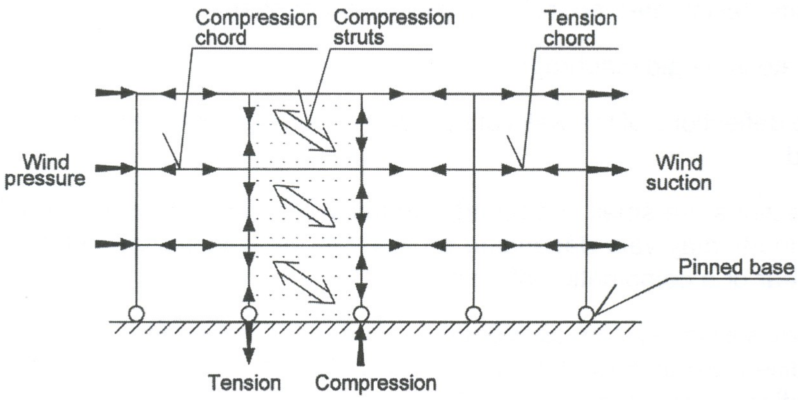
Figure 2.42 Effective Heights Of Plain Walls

The slenderness limits for reinforced concrete walls are given in Part 1, Table 3.25 of the Code. The effective heights are determined by similar methods used for columns and in accordance with Part 1, clause 3.8, Tables 3.21 and 3.22, of the Code. When the beams and slabs transmitting forces into the reinforced concrete walls are simply supported, the effective heights of the walls are assessed similarly for plain walls. These are shown in Figure 2.42 above.

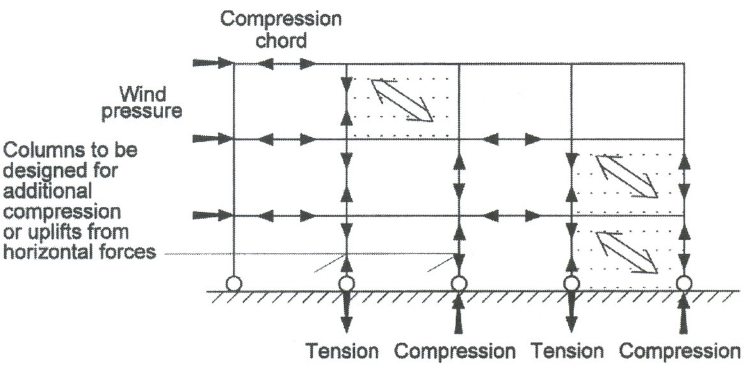
2.5.3 Distribution of horizontal loads

Due to their large in-plane stiffness and strength, precast concrete walls offer the best solution to stability irrespective of the number of storeys in a multi-storey construction.

The horizontal forces, which consist of the greater of either the ultimate wind forces or 1.5% of the characteristic dead weight of the structure are transferred by diaphragm action of the floor to the stability walls in the manner shown in Figure 2.43.



(a) Continuous Wall In Vertical Plane



(b) Walls In Different Vertical Planes

Figure 2.43 Horizontal Load Transfer In Braced Structure

It may be necessary to consider uplift particularly in gable end walls which do not carry any significant vertical loads.

The distribution of horizontal loads between stability wall elements is generally determined by the position and stiffness of the walls in the structure. When the location and distribution of the walls are such that their centre of resistance coincides closely with the centre of the mass and the geometric centroid of the completed building, the distribution of horizontal loads is proportional to their flexural stiffness. If the walls are more squat, i.e., its height to length is less than three, shear deflection will govern and the load distribution will be a function of the shear stiffness of the wall elements. If the Young's modulus and shear modulus are similar in all the walls, the stiffness of each wall element is then proportional to its second moment area, I, of the uncracked section or to the web cross sectional area, generally taken as 80% of the total web area if the wall deflection is predominantly due to shear.

In plain concrete walls, the Code stipulates in Part 1 clause 3.9.4.7, that when the resultant eccentricity from a horizontal force is greater than 1/3 of the length of the wall, the stiffness of the wall will be ignored and the horizontal forces adjusted to be carried by the remaining walls.

In majority of the buildings, the walls are normally non-symmetrical and torsional stability may need to be considered in the design. The torsions in the walls in a complicated layout can be determined using computer software with appropriate structural modelling. In relatively straight forward cases, it suffices just to carry out approximate and simple manual calculations to determine the horizontal loads due to torsion effect.

The approximate design method (reference 6) assumes that :

1. the floor plate is a rigid diaphragm,
2. the relative deflections of the walls are proportional to the distances from the centroid of flexural rigidity, and
3. shear deflections are small compared with flexural deflections even though the distribution of horizontal loads may vary depending on whether the wall system behaviour is predominantly flexure, shear or a combination of both.

Figure 2.44 shows a wall system containing n numbers of wall profiles. The centroid of the stiffness or flexural rigidities (EI) of the wall system is calculated from an arbitrary reference point O in a x-y cartesian coordinate system. If the Young's modulus E is similar in all the walls, the flexural rigidity of each wall element is then proportional to its second moment of area I.

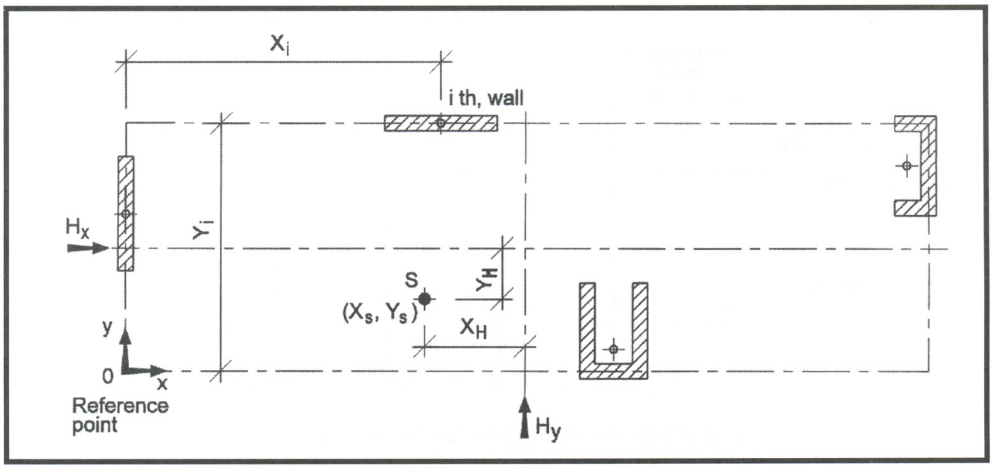


Figure 2.44 Notations Of Wall System Subjected To Torsion

The total stiffness of the wall system in the two principal directions is determined by :

$$I_\chi = \Sigma I_{i\chi} \qquad I_y = \Sigma I_{iy}$$

The centre of the shear force of the wall system is :

$$\chi_s = \Sigma(I_{i\chi}\, \chi_i)/I_\chi$$

$$y_s = \Sigma(I_{iy}\, y_i)/I_y$$

The resultant horizontal loads denoted by H_χ and H_y act at a distance χ_H, y_H from the centroid of the shear force.

The torsion, taking positive in anti-clockwise direction, is given by

$$T = H_y\, \chi_H - H_\chi\, y_H$$

The torsional strength of the individual wall system can be determined by means of the formula :

$$t = \Sigma I_{i\chi}\, (\chi_i - \chi_s)^2 + \Sigma I_{iy}\, (y_i - y_s)^2$$

The horizontal load on each wall section can then be calculated by means of the components in the χ and y directions.

$$H_{i\chi} = I_{iy} [H_\chi/I_y - T(y_i - y_s)/t]$$
$$H_{iy} = I_{i\chi} [H_y/I_\chi + T(\chi_i - \chi_s)/t]$$

It can be observed from the above that in symmetrical wall system where T = 0, the distribution of horizontal loads in the wall sections will then be directly proportional to the moment of inertia of the individual walls.

Design Example 8 is used to illustrate the design method outlined above.

2.5.4 Infill Precast Walls in Skeletal Frame Structure

Precast concrete walls are commonly used as infill walls between framing elements in a skeletal frame structure to function as stability walls. The walls are assumed not to carry any building loads and that beams between the wall panels are considered as separate structural elements even though the gap between them is grouted solidly. The walls are also assumed to be free from simultaneous in-plane and out-of-plane wind loadings.

The infill walls are generally designed as plain walls with sufficient reinforcement to resist diagonal cracking in the panel and to maintain the intrinsic shape of the panel particularly at the corners. The tensile strength of the concrete is ignored in the design.

The behaviour of infill concrete walls and general design principles subjected to horizontal loads are outlined in reference 7 and illustrated in Figure 2.45.

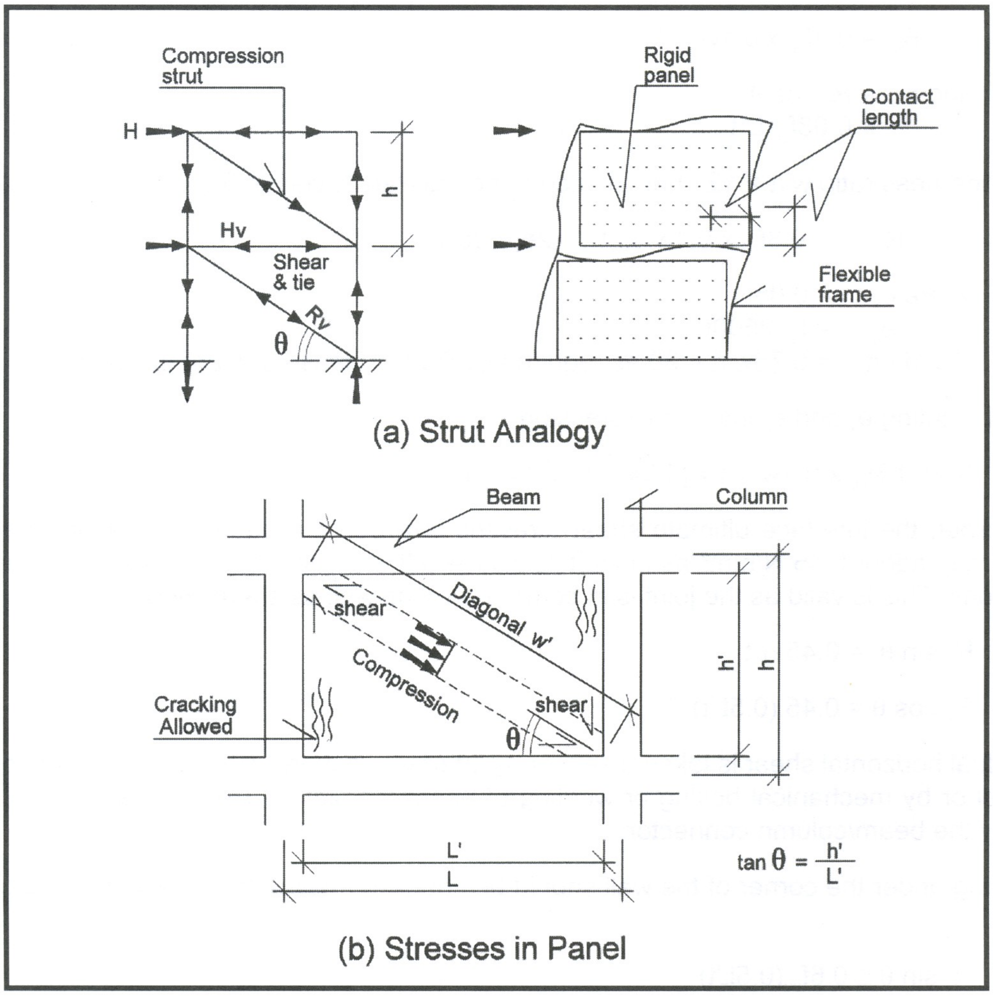


Figure 2.45 Infill Wall Panel Design Principles (reference 7)

The ultimate horizontal forces are resisted by a diagonal compressive strut across the wall panel. As the flexible frame deforms under horizontal loadings, compressive forces are transmitted at the two locked corners of the panels by interface shear stresses. The transmission of forces over the diagonal compressive strut at the corners of the panel occurs over an extended contact length. This gives rise to the effective width of the compressive strut. The width of the compressive strut is also dependent on the relative stiffness of the wall panel and the framing structure, and on the aspect ratio of the panel. An upper bound width of 0.1w' may generally be used in the design together with a consideration of strength reduction due to slenderness effect.

The contact lengths between panel and framing elements may be taken as

a. with beams = 0.5L'
b. with column : $\alpha = \pi/2\lambda$

and $\lambda = 4\sqrt{(E_i \, t \tan 2\theta)/(4E_f \, I \, h')}$

where E_i, E_f = modulus for infill and frame respectively

t = thickness of infill wall

I = moment of inertia of beams or columns whichever is lesser

h' = net height of infill panel

θ = slope of infill

The infill wall is considered braced as it is constructed on all sides. The effective length of the diagonal strut is then given by $l_e = 0.75w'$ where w' is the length of the diagonal strut. The ultimate concrete compression is assumed to be $0.3f_{cu}$ as in plain wall.

From Figure 2.45, the strength of the compressive strut, R_v, is given by:

$$R_v = 0.3f_{cu} \times 0.1w' \times t$$

$$\text{and } H_v = R_v \cos\theta = 0.03f_{cu} \, L' \, t$$

If the slenderness ratio w'/t > 12, then in accordance to Part 1, clause 3.9.4.15 of the Code,

$$R_v = 0.3f_{cu} \times 0.1w' \times (t - 1.2e_x - 2e_a)$$

where $e_x = 0.05t$

$e_a = l_e^2/2500t$

and $l_e/t = 0.75w'/t < 30$ as required by Part 1, clause 3.9.4 of the Code

Substituting e_x and e_a into the expression

$$R_v = 0.3f_{cu} \times 0.1w' \times t \times [0.94 - (l_e/t)^2/1250]$$

At the corner, the interface ultimate shear stresses between the panel and framing beams and columns may adopt 0.45 N/mm^2 as in Part 1, clause 5.3.7 of the Code for plain surfaces without castellations. This is valid as the joint is in compression as well as shear. Hence,

$$R_v \sin\theta = 0.45 \, \alpha \, t$$

$$R_v \cos\theta = 0.45 \, (0.5L't)$$

Any residual horizontal shear is taken up either by interface reinforcement anchored into beam and wall panel or by mechanical bolting or welding of anchored plates. The vertical residual shear is carried by the beam/column connector.

The bearing under the corner of the wall should be checked against the weakest interface jointing concrete by

$$R_v \sin\theta \leq 0.6f_{cu}(0.5L't)$$

The design of infill plain walls in skeletal frame structure is illustrated in Design Example 9.

2.5.5 Cantilever precast concrete walls

Cantilever precast concrete walls are usually provided as enclosures for staircases and lift shafts. They are designed as conventional reinforced concrete walls with special emphasis on the connection details of vertical and horizontal joints of the walls for the transfer of compression, tension and shear forces.

The walls may fail in one of the following modes as shown in Figure 2.46.

a. Shear slip at the horizontal and vertical joints if the walls are assembled from separate units and when L/h is greater than three.

b. Flexural tension failure under large overturning moment and lightly loaded situation.

c. Flexural compression failure due to combined axial forces and moments.

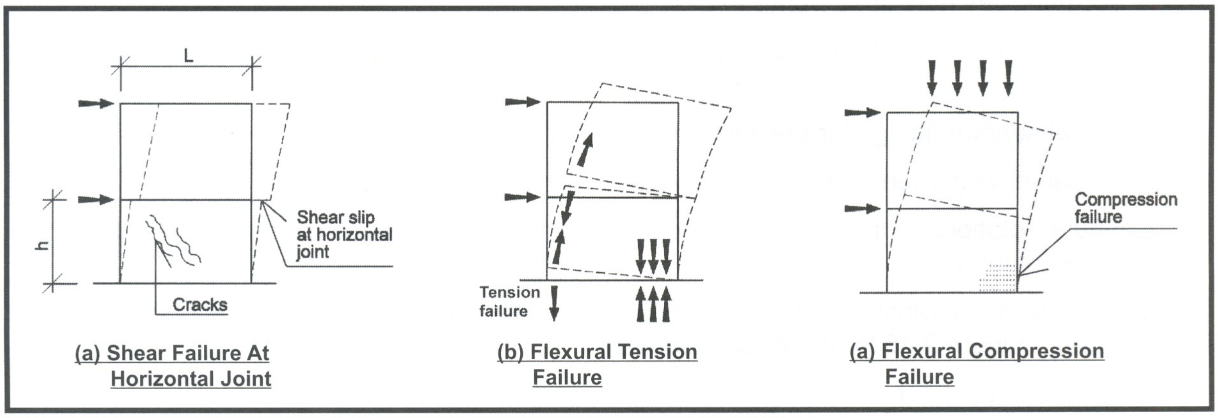


Figure 2.46 Failure Mode Of Cantilever Precast Concrete Walls

The design of the wall is carried out at the ultimate limit state as illustrated in Figure 2.47.

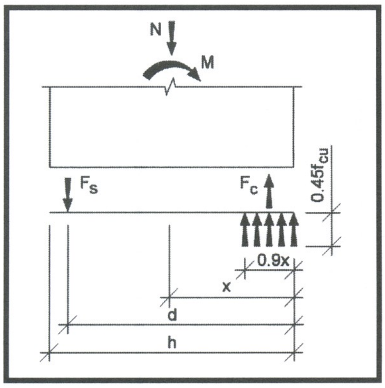


Figure 2.47 Design Principle Of Cantilever Precast Concrete Walls

The concrete compression forces under combined action of axial load and moment are given by

$$F_c = 0.45f_{cu} \times 0.9\chi \times t$$

Under axial loads

$$N = F_c - F_s$$

where F_s is the tensile resistance provided by reinforcement

$$F_s = 0.87f_yA_s$$

Hence $N = 0.45f_{cu} \times 0.9\chi \times t - 0.87f_yA_s$

and

$$\begin{aligned} M &= F_c(d - 0.45\chi) - N(d - h/2) \\ &= 0.45f_{cu} \times 0.9\chi \times t(d - 0.45\chi) - N(d - h/2) \\ &= 0.405f_{cu}\,\chi\, t(d - 0.45\chi) - N(d - h/2) \end{aligned}$$

To transfer tension force F_s across the joint, isolated connections using grouted pipe sleeves, bolting and welding may be used.

2.5.6 Vertical load capacity of precast walls

One of the primary structural functions of walls is to carry the vertical loads to the foundation. From a structural viewpoint, the walls can be regarded as columns of unit length. The theoretical calculations of such columns under axial load or axial load with eccentricities are dealt with in most standard literature on structural design and a set of column design charts can be found in BS8110 : Part 3. These column design charts can be applied to the design of in-situ as well as precast reinforced walls.

In low- to medium-rise buildings, plain precast walls may be used due to the simplicity in the wall connections as well as the inherent large carrying capacity of the walls based on concrete alone. However, as a result of the lack of structural reinforcement in plain walls, the vertical load capacity can be reduced drastically if the wall is subjected to excessive flexural tensile stresses resulting from transverse load eccentricities.

These eccentricities may arise from :

1. the floor elements with unequal span or with different design loadings on each side of the wall,
2. special support fittings for the floor elements,
3. vertical misalignment or tilting in the vertical plane of the wall panels
4. angular rotations of the ends of the floor element which introduce moments at the top and bottom of the walls.

The magnitude of the transverse load eccentricities is given in Part 1, clause. 3.9.4, of the Code and is illustrated in Figure 2.48 for the following cases :

1. loads from the floor
2. loads from special support fitting
3. resultant loads

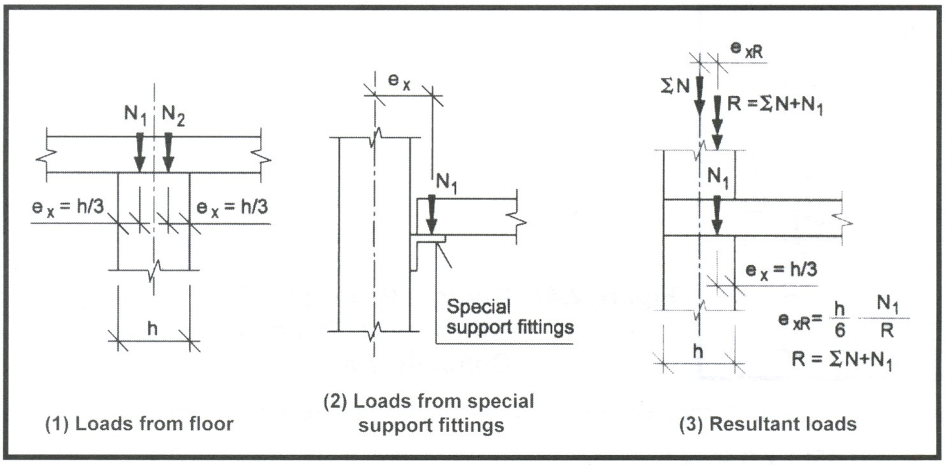


Figure 2.48 Design Load Eccentricities In Walls

For an unbraced wall, the eccentricities of all vertical loads and the moments due to lateral loads are considered when calculating the resultant eccentricity.

Minimum design transverse eccentricity is t/20 or 20 mm, whichever is greater.

The vertical load carrying capacity of plain walls is calculated using the following equations :

1. Stocky braced wall

   $n_w \leq 0.3f_{cu}(t - 2e_\chi)$

2. Slender braced wall

   $n_w \leq 0.3f_{cu}(t - 1.2e_\chi - 2e_a)$

3. Unbraced wall

a. $n_w \leq 0.3f_{cu}(t - 2e_{\chi,1})$ or

b. $n_w \leq 0.3f_{cu}(t - 2e_{\chi,2} - 2e_a)$

whichever is smaller.

where e_χ = actual resulting transverse load eccentricity t/20 or 20 mm, whichever is greater.

$e_{\chi,1}$, $e_{\chi,2}$ = resulting transverse load eccentricity at the top and bottom of the wall respectively, as in e_χ

e_a = additional deflection due to slenderness effect
= $l_e^2/2500t$ where l_e is the effective height of the wall

2.5.7 Design charts

The design charts for stocky and slender braced plain walls are shown in Figures 2.49 and 2.50 respectively. The walls are assumed to be pin-connected at the floor levels.

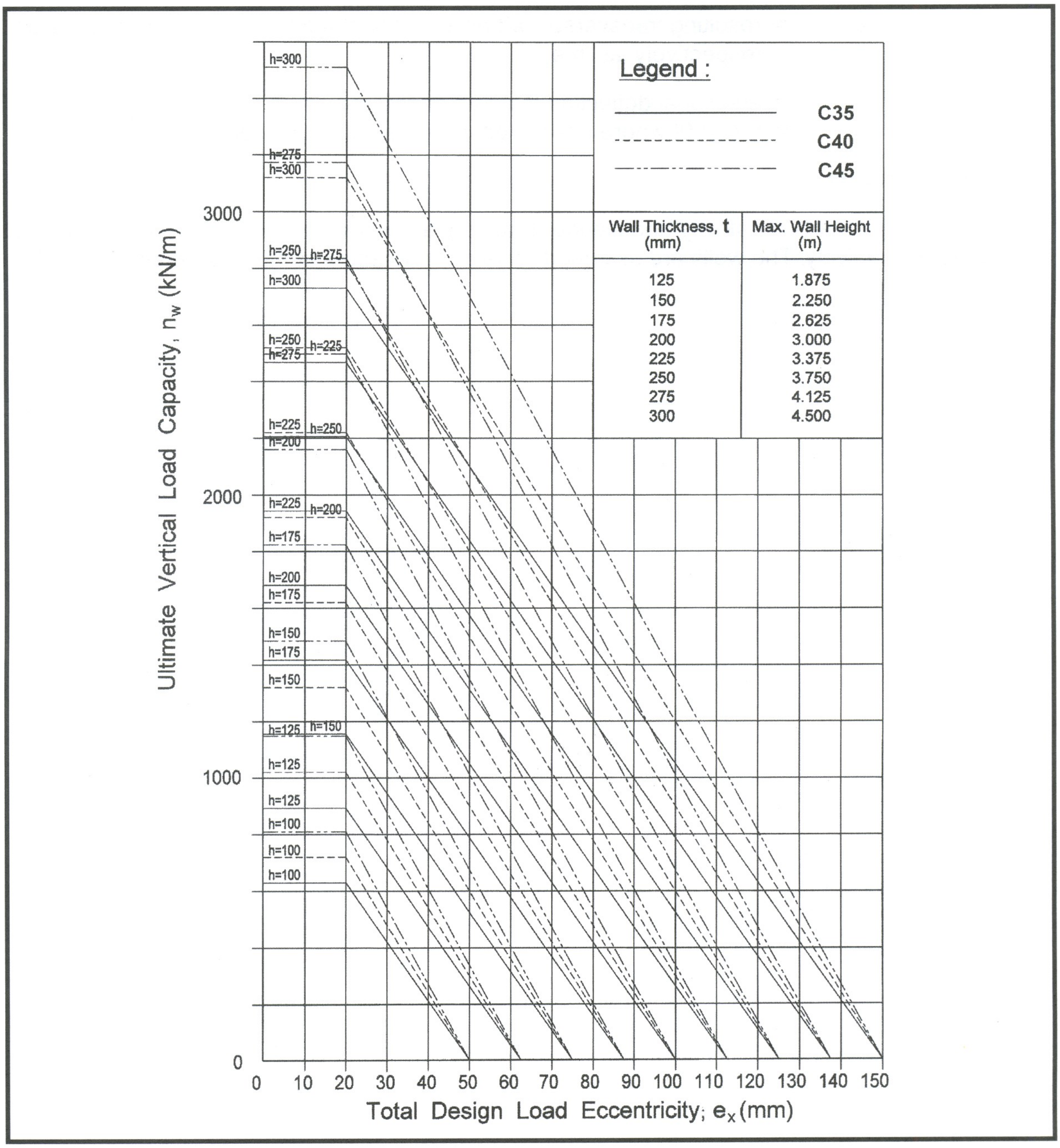
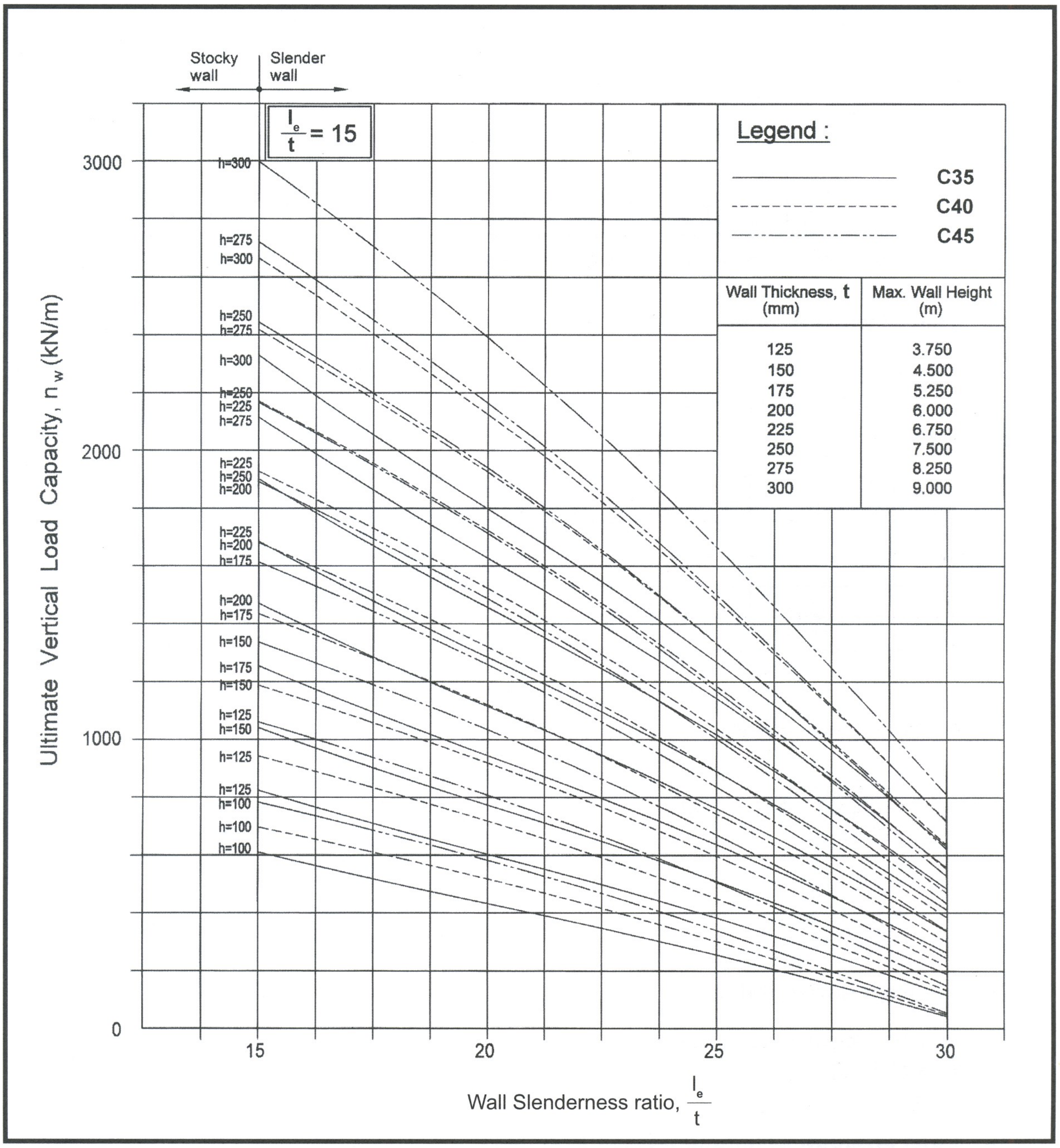


Figure 2.49 Ultimate Vertical Load Capacity (kN) Of Plain Stocky Braced Walls ($l_e/h \leq 15$)

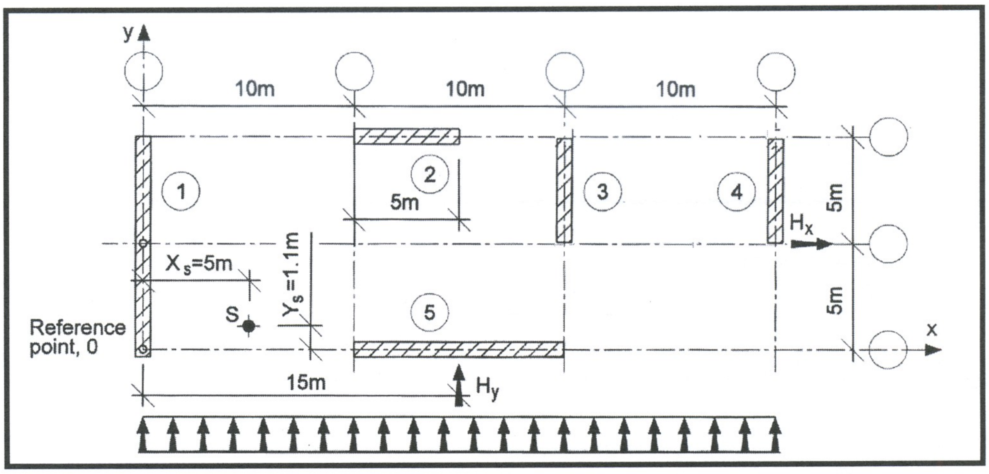


Wall Thickness, t (mm)	Max. Wall Height (m)
125	3.750
150	4.500
175	5.250
200	6.000
225	6.750
250	7.500
275	8.250
300	9.000

Figure 2.50 Ultimate Vertical Load Capacity Of Braced Slender Walls ($15<l_e/h\leq30$)

2.5.8 Design examples

Design Example 8 : Distribution of Horizontal Loads



Note : All walls to be of the same thickness

1. **Centroid of wall stiffness**

   Using a 5 m long wall as reference and with a moment of inertia, I, the stiffness of walls 1 to 5 and the centroid of wall stiffness are calculated as below:

Wall	χ_i (m)	y_i (m)	I_χ	I_y
1	0	5.0	8I	0
2	12.5	10.0	0	I
3	20.0	7.5	I	0
4	30.0	7.5	I	0
5	15.0	0	0	8I
Total			10I	9I

The centroid of wall stiffness

$\chi_s = \Sigma(I_{i\chi}\, \chi_i)/I_\chi$
$= (I \times 20 + I \times 30)/10I$
$= 5$ m

$y_s = \Sigma(I_{iy}\, y_i)/I_y$
$= I \times 10/9I$
$= 1.1$ m

2. **Torsion from horizontal load**

   Taking anti-clockwise as positive
   $T = H_y \times (15 - 5)$
   $= 10H_y$

3. **Torsional strength of individual wall sections**

   $t = \Sigma I_{i\chi}\, (\chi_i - \chi_s)^2 + \Sigma I_{iy}\, (y_i - y_s)^2$
   $= 8I \times 5^2 + I \times 15^2 + I \times 25^2 + I \times 8.9^2 + 8I \times 1.1^2$
   $= 1138.9I$

4. Distribution of horizontal forces

$H_{i\chi} = I_{iy} [H_\chi/I_y - T(y_i - y_s)/t)]$

$H_{iy} = I_{i\chi} [H_y/I_\chi + T(\chi_i - \chi_s)/t)]$

Substituting $H_\chi = 0$, $\Sigma I_y = 9I$, $\Sigma I_\chi = 10I$, $T = 10H_y$, $t = 1138.9I$, $\chi_s = 5$, $y_s = 1.1$

Hence :

$H_{i\chi} = -8.78 \times 10^{-3}(y_i - 1.1) \times H_y \times I_{iy} / I$

$H_{iy} = [0.1 + 8.78 \times 10^{-3}(\chi_i - 5)] \times H_y \times I_{ix} / I$

Wall 1 : $H_{1\chi} = 0$, $H_{1y} = +0.45H_y$

Wall 2 : $H_{2\chi} = -0.08H_y$, $H_{2y} = 0$

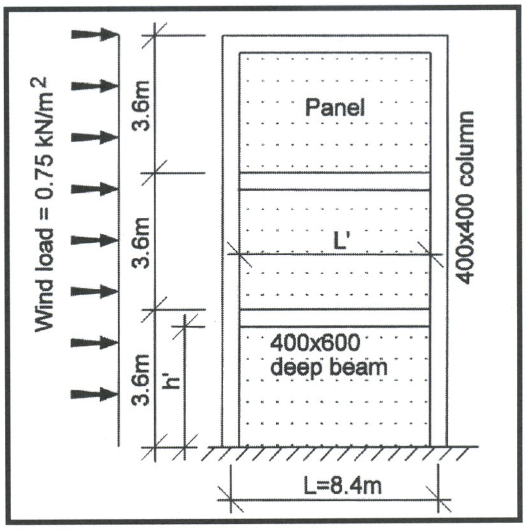
Wall 3 : $H_{3\chi} = 0$, $H_{3y} = +0.23H_y$

Wall 4 : $H_{4\chi} = 0$, $H_{4y} = +0.32H_y$

Wall 5 : $H_{5\chi} = +0.08H_y$, $H_{5y} = 0$

Design Example 9 : Infill Precast Walls

The building is 60 m long and is braced at both ends with infill walls between the framing beams and columns. Determine the minimum thickness of the infill precast concrete walls at the first storey to provide stability to the building. Use f_{cu} = 35 N/mm^2 for the walls and frame. Design wind pressure = 0.75 kN/m^2.



Gable End Walls

L' = 8.4 - 0.4 = 8.0 m

h' = 3.6 - 0.6 = 3.0 m

w' = $\sqrt{(8.0^2 + 3.0^2)}$ = 8.544 m

θ = $\tan^{-1}$ 3/8.544 = 19.35^0

1. **Maximum ultimate horizontal forces acting at second storey per frame**

H = 1.4 x 0.75 x (3.6 + 3.6 + 3.6/2) x 60/2
= 283.5 kN

R_v = H/cos θ
= 300.5 kN

Compression capacity of diagonal strut

R_v = $0.3f_{cu}$ x 0.1w' x t[0.94 - $(l_e/t)^2$/1250]
= 0.3 x 35 x 0.1 x 8544 x t[0.94 - $(6408/t)^2$/1250] x 10^{-3} (in kN)
= 8.43t - 2.947 x 10^5/t

Minimum wall thickness required 300.5 = 8.43t - 2.947 x 10^5/t

Solving the quadratic equation t = 205.6 mm

Slenderness ratio = 0.75w'/t
= 0.75 x 8544/205.6
= 31.2 > 30

Use 0.75w'/t = 30
t = 214 mm

Say t = 225mm

2. Check interface shear stress

Column Interface $\lambda = \sqrt[4]{(E_i\, t \tan 2\theta)/(4E_f\, I\, h')}$

$E_i = E_f = 27$ kN/mm^2

$I_{col} = 400 \times 400^3/12$
$= 2.133 \times 10^9$ mm^4

$t = 225$ mm
$h' = 3.0$ m
$\lambda = 1.629 \times 10^{-3}$
$\alpha/h = \pi/2\lambda h'$
$= \pi/(2 \times 1.629 \times 10^{-3} \times 3000)$
$= 0.321$

Contact length $\alpha = 0.321 \times 3000$
$= 964.2$ mm

$R_v \sin\theta = 0.45\alpha t$
$R_v = 0.45 \times 964.2 \times 225 \times 10^{-3}/\sin 19.35^0$
$= 294.6$ kN < 300.5 kN

Residual vertical shear to be resisted by beam/column connection
$= (300.5 - 294.6)\sin 19.35^0$
$= 1.95$ kN (not critical)

3. Beam Interface

$R_v \cos\theta = 0.45 \times 0.5L't$

$R_v = 0.45 \times 0.5 \times 8000 \times 225 \times 10^{-3}/\cos 19.35^0$
$= 429.2$ kN > 300.5 kN OK

Use 225 mm thick plain precast wall

4. Check Bearing Stress

$R_v \sin\theta < 0.6f_{cu}(0.5L't)$

$R_v \sin\theta = 300.5 \sin 19.35^0$
$= 99.6$kN

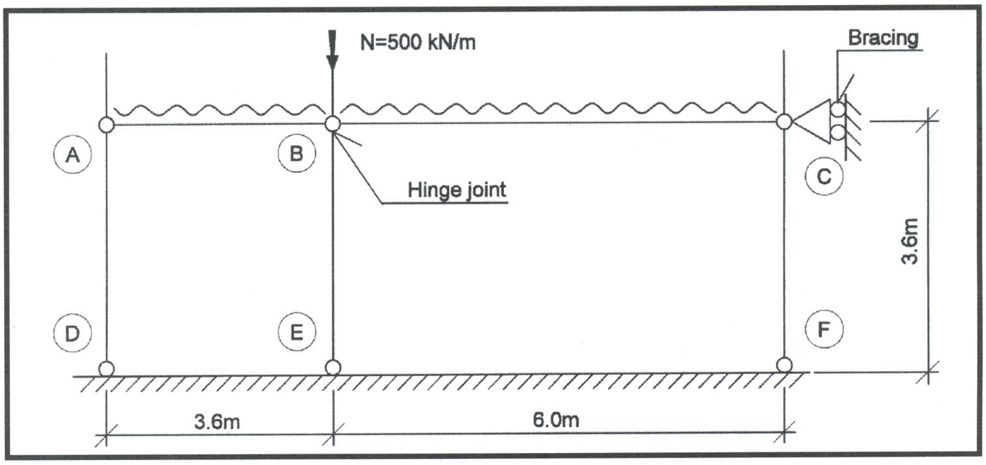
$0.6f_{cu}(0.5L't) = 0.6 \times 35 \times 0.5 \times 8000 \times 225 \times 10^{-3}$
$= 18900$ kN > 99.6 kN OK

225mm thick plain precast wall is adequate

Design Example 10 : Braced Load Bearing Plain Walls

Check whether the internal 150 mm thick plain wall BE at the first storey in a braced building is adequate to carry an ultimate wall load of 500 kN/m and the floor loads shown in the figure below. Design concrete strength is f_{cu} = 35 N/mm^2.

Design dead load (unfactored) = 6.0 kN/m^2
live load (unfactored) = 4.0 kN/m^2



1. Calculate ultimate design load (per metre length)

Maximum design ultimate load = 1.4 x 6.0 + 1.6 x 4.0
= 14.8 kN/m

Minimum design ultimate load = 1.0 x 6.0
= 6.0 kN/m

2. Vertical floor reactions

For span AB : maximum floor reaction = 14.8 x 3.6/2
= 26.6 kN/m

minimum floor reaction = 6.0 x 3.6/2
= 10.8 kN/m

For span BC : maximum floor reaction = 14.8 x 6.0/2
= 44.4 kN/m

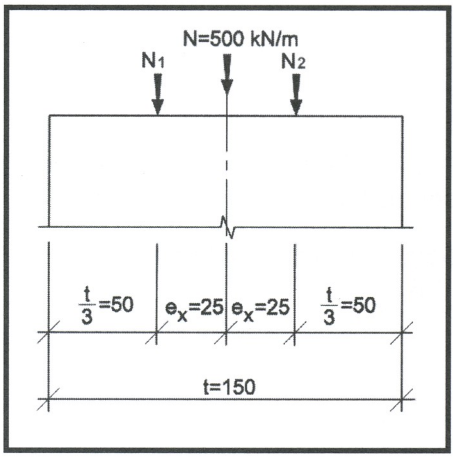
minimum floor reaction = 6.0 x 6.0/2
= 18.0 kN/m

3. Load eccentricities

For span AB : N_1 = maximum reaction = 26.6 kN/m
minimum reaction = 10.8 kN/m

For span BC : N_2 = maximum reaction = 44.4 kN/m
minimum reaction = 18.0 kN/m

$e_x = t/20 = 7.5 \text{ mm} < 20 \text{ mm}$
Use $e_x = 20$ mm



Case 1 : maximum floor load results in minimum load eccentricity

$$e_x = \frac{t}{6} \; \frac{N_2 - N_1}{N + \Sigma(N_1 + N_2)}$$

$$= \frac{150}{6} \; \frac{(44.4 - 26.6)}{500 + (44.6 + 26.6)}$$

$$= 0.78 \text{ mm} < 20 \text{ mm}$$

Use $e_x = 20$ mm

Case 2 : maximum load in span BC, minimum load in span AB, result in maximum load eccentricity

$$e_x = \frac{t}{6} \; \frac{N_2 - N_1}{N + \Sigma(N_1 + N_2)}$$

$$= \frac{150}{6} \; \frac{(44.4 - 10.8)}{500 + (44.6 + 10.8)}$$

$$= 1.5 \text{ mm} < 20 \text{ mm}$$

Use $e_x = 20$ mm

Eccentricity from slenderness effect

Effective wall height = 1.0 x 3.6
= 3.6 m

l_e /h = 3600/150
= 24 > 15 for a braced wall

Hence, additional eccentricity e_a due to slenderness effect has to be considered.

e_a = l_e^2 / 2500t
= 3600^2 / 2500 x 150
= 34.6 mm

4. Total load eccentricities

Load eccentricity from floor is 20 mm for both cases 1 and 2. Hence the wall is designed using the following loads and load eccentricities :

ΣN = 500 + 44.4 + 26.6 + self weight of wall at mid-storey height
= 571.0 + 1.4 x 0.15 x 24 x 1.8
= 580.1 kN/m

e_x = 20 mm
e_a = 34.6 mm

5. Vertical load carrying capacity

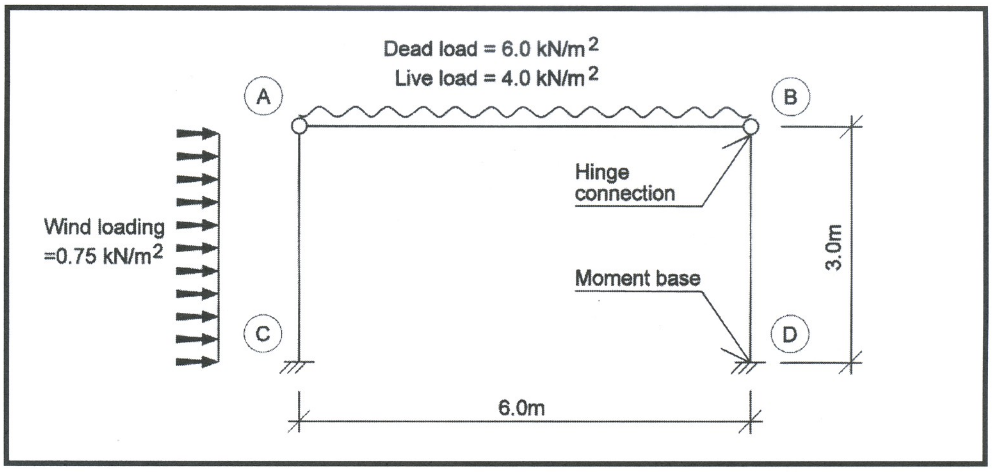
For braced slender plain walls :

$n_w \leq 0.3f_{cu}(t - 1.2e_x - 2e_a)$
$\leq$ 0.3 x 35(150 - 1.2 x 20 - 2 x 34.6)
$\leq$ 596.4 kN/m > 580.1 kN/m

150 mm thick plain wall is adequate

Design Example 11 : Unbraced Load Bearing Plain Walls

Design the 250 mm thick wall AC in a single storey, unbraced building for the floor loading and lateral wind load shown in the figure below. The design concrete strength is f_{cu} = 35 N/mm^2.



1. **Calculate ultimate design vertical load (per metre length)**

Case 1 : maximum design load = 1.4 x 6.0 + 1.6 x 4.0
= 14.8 kN/m

Case 2 : Dead and wind = 1.4 x 6.0
= 8.40 kN/m

Case 3 : Dead, live and wind = 1.2(6.0 + 4.0)
= 12.0 kN/m

2. **Vertical load and moment at bottom of wall**

Wind moment is shared between two walls.
Moment per wall = 0.75 x 3^2 x 0.5/2
= 1.7 kNm/m

Case 1 : Vertical load = 14.8 x 6/2 + self weight of wall
= 44.4 + 1.4 x 0.25 x 24 x 3.0
= 69.6 kN/m

Wind moment = 0

Case 2 : Vertical load = 8.4 x 6/2 + self weight of wall
= 25.2 + 1.4 x 0.25 x 24 x 3.0
= 50.4 kN/m

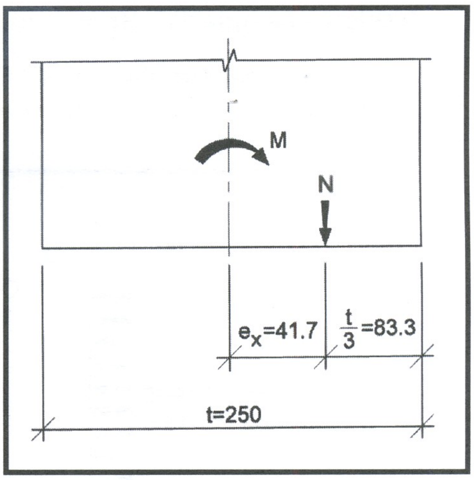
Wind moment = 1.4 x 1.7
= 2.4 kNm/m

Case 3 : Vertical load = 12.0 x 6/2 + self weight of wall
= 36.0 + 1.2 x 0.25 x 24 x 3.0
= 57.6 kN/m

Wind moment = 1.2 x 1.7
= 2.0 kNm/m

3. Load eccentricities

a. Vertical load and wind moment



Vertical Load And Wind Moment

Case 1 : $e_x = 41.7$ mm

Case 2 : Total moment $= Ne_x + 2.4$
$= 50.4 \times 0.0417 + 2.4$
$= 4.5$ kNm/m

Resultant eccentricity $e_{x2} = M/N$
$= 4.5 \times 10^3/50.4$
$= 89.3$ mm

Case 3 : Total moment $= Ne_x + 2.0$
$= 57.6 \times 0.0417 + 2.0$
$= 4.4$ kNm/m

Resultant eccentricity $e_{x2} = M/N'$
$= 4.4 \times 10^3/57.6$
$= 76.4$ mm

b. Slenderness effect

Effective wall height $l_e = 1.5l_o$
$l_o = 3.0$ m
$l_e = 1.5 \times 3.0$
$= 4.5$ m

Additional eccentricity $e_a = l_e^2/2500t$
$= 4500^2/(2500 \times 250)$
$= 32.4$ mm

4. Vertical load carrying capacity

The vertical load carrying capacity of the unbraced concrete plain wall is the smaller of the following:

a. $n_w \leq 0.3f_{cu}(t - 2e_{x1})$
b. $n_w \leq 0.3f_{cu}(t - 2e_{x2} - 2e_a)$

By inspection, the load carrying capacity is given by the second expression which is applied at the bottom of the wall.

Case 1 : $e_{x2} = 41.7$ mm $e_a = 32.4$ mm
$n_w \leq 0.3 \times 35(250 - 2 \times 41.7 - 2 \times 32.4)$
≤ 1068.9 kN/m > 69.6 kN/m OK

Case 2 : $e_{x2} = 89.3$ mm $e_a = 32.4$ mm
$n_w \leq 0.3 \times 35(250 - 2 \times 89.3 - 2 \times 32.4)$
≤ 69.3 kN/m > 50.4 kN/m OK

Case 3 : $e_{x2} = 76.4$ mm $e_a = 32.4$ mm
$n_w \leq 0.3 \times 35(250 - 2 \times 76.4 - 2 \times 32.4)$
≤ 340.2 kN/m > 57.6 kN/m OK

250 mm thick plain wall is adequate.